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On the regularized modeling of density currents

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Abstract. In this work we studied four regularization models with deconvolution for density currents, namely, Boussinesq-α, Boussinesq-ω, Boussinesq-Leray and Modified-Boussinesq-Leray. A Crank-Nicolson in time and Finite Element in space algorithm is proposed and proved to be unconditionally stable and optimally convergent, which is also verified through convergence rates in computational simulations. Finally, the models are compared through the Marsigli's flow problem. We found that all regularization models produced accurate solutions for low Reynolds number. Moreover, as expected, we observe that increasing deconvolution order improves solution. On the other hand, for high Reynolds number Boussinesq-Leray and Boussinesq-α with deconvolution produced the most accurate solutions. However, from the computational viewpoint, the Boussinesq-Leray model presented advantage due to its decoupling between momentum and filter equations which permits to increase the deconvolution order with no significant increase in the computational cost.

Keywords. Regularized modeling, density current, Boussinesq model.

1 Introduction

Density currents are one of the most important type of geophysical flow, such as atmospheric and oceanic flows. Moreover, density currents are also quite important for engineering applications. They can be defined as flows forced by the buoyant force due to density gradients in the fluid.

One of the most used models to represent density currents is the Boussinesq model:

$$\begin{aligned} \tfrac{\partial u}{\partial t} - u \times \nabla \times u - Re^{-1}\Delta u + \nabla p - RiT\hat{k} &= f \text{ in } \Omega \\ \nabla \cdot u &= 0 \text{ in } \Omega \\ \tfrac{\partial T}{\partial t} - u \cdot \nabla T - (RePr)^{-1}\Delta T &= 0 \text{ in } \Omega \end{aligned}$$

where Ω is the domain, u is the velocity, p is pressure, T is temperature, f is a given force, Re is the Reynolds number, Ri is the Richardson number, Pr is the Prandtl number.

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In this work, regularization models are studied in order to reduce computational cost in simulations with the Boussinesq model. Regularization models were initially developed for theoretical purposes, but recently considerable interest has risen for regularization models as simpler models to deal with turbulence in numerical simulations with Navier-Stokes equations. In this approach, the regularization model is obtained from the direct regularization of the convective flux. The main advantage of this kind of model in relation to Direct Numerical Simulation (DNS) is that the number of degrees of freedom in simulations are significant fewer in regularization models. Here we have studied four regularizations for Boussinesq model, namely: alpha [3], omega [3], Leray [2] and Modified-Leray [1] and their respectively deconvolution versions.

2 Preliminary Results

First of all, we propose a linearized Crank-Nicolson in time and Finite Element in space scheme for the regularized Boussinesq models. Then, we prove they are unconditionally stable and optimally convergent and show that increasing deconvolution order increase the consistency error order of the regularized model.

Moreover, we tested the proposed algorithms in numerical simulations. Initially, we showed that the convergence rates estimated in simulations using a known analytical solution agrees well with the theoretical ones. Afterwards, we compared the regularized models in the Marsigli's density flow problem. In this experiment, we observed that for low Reynolds numbers, all tested regularized models produced similar solutions, being more accurate for larger deconvolution order. On the other hand, for high Reynolds number we observed that, despite of all first order deconvolved models being better than non-regularized Boussinesq model with and without artificial viscosity, Boussinesq-Leray and Boussinesq-α produced the best solutions. However, when comparing Boussinesq-α and Boussinesq-Leray, the latter presented a remarkable computational advantage due to its decoupling between momentum and filter equations which permits the Boussinesq-Leray to work with larger deconvolution order while maintaining a viable computational cost.

References

[1] A. Ilyin, E. Lunasin and E. Titi, A modified-Leray-α subgrid scale model of turbulence, Nonlinearity, vol. 19, 879-897, (2006), DOI: 10.1088/0951-7715/19/4/006.

[2] W. Layton, C. Manica, M. Neda and L. Rebholz, Numerical analysis and computational testing of a high accuracy Leray-deconvolution model of turbulence, Num. Met. for Partial Diff. Equations, vol. 24, 555-582, (2008), DOI: 10.1002/num.20281.

[3] W. Layton, C. Manica, M. Neda and L. Rebholz, Numerical analysis and computational comparisons of the NS-alpha and NS-omega regularizations, Comp. Met. in Applied Mech. and Eng., vol. 199, 916-931, (2010), DOI: 10.1016/j.cma.2009.01.011.