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Optimal Power Flow Problem Using Interior-Point Method with Block Structure

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Abstract This work focuses on optimizing power flow in the electrical sector, considering uncertainties in energy demand, to minimize costs and energy loss, within the constraints of the physical network. We use a two-stage stochastic programming, creating discrete scenarios associated with the uncertainty of demand. The problem is solved using MSSO-BlockIP, a specialized software employing the Interior-Point Method for stochastic programming. The work presents the mathematical formulation using splitting variables and provides computational results, comparing this approach with a traditional one. The findings indicate that the proposed method yields solutions consistent with Gurobi, achieving notable improvements in terms of running time and the number of iterations.

Keywords. Stochastic Programming, Power Flow, Interior-Point Methods, Splitting Variables.

1 Introduction

Optimization of power flow is important for the efficiency of the electrical sector, impacting generation costs, transmission reliability, energy production, and ensuring operational safety. Uncertainty is important in this context, influencing both short and long-term plans.

This work exploits how uncertainty in energy demand affects the determination of the ideal power flow. We focus on finding an effective method to minimize the costs and energy loss associated with power flow that admits demand fluctuations, considering the physical constraints of the electrical network. This problem is crucial for the efficiency of the electrical sector, directly impacting costs, minimizing power losses, and affecting planning.

Two-stage stochastic programming is a feasible approach to handle demand uncertainty [8]. This methodology allows for creating discrete scenarios with their associated probabilities, providing a robust framework to address the complexity of demand variations, unlike the non-stochastic approach that does not account for randomness.

Our methodological strategy involves splitting the variables to create blocks associated with discrete demand scenarios and stages, aiming to build a constraint matrix by blocks. This process will be conducted using computational methods optimized for this type of structure, employing the Interior-Point method for quadratic problems, enabling the resolution of the optimal power flow problem even under conditions of uncertainty in energy demand.

The remainder of this work is organized as follows. Section 2 presents the pre-dispatch problem and its two-stage stochastic extension for demand uncertainty, along with the block structure used for its solution. Section 3 provides computational results, and Section 4 concludes.

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2 Optimal Power Flow

The operations of energy systems require the use of specialized methods to optimize resource utilization, aiming to minimize costs and losses in energy generation and transmission. Additionally, the system must be prepared to handle unexpected events, as well as variations in energy demand throughout the day and the climatic season of the year. During the pre-dispatch of hydroelectric systems, power plants must meet established goals through long-term planning. Knowing this, we seek to fulfill the requirements of uncertain demand, minimizing thermal generation costs and conserving water resources.

The Optimal Power Flow problem is an optimization problem in the energy sector that aims to optimize an objective function such as economic dispatch, subject to constraints reflecting operational limits and physical laws of the electrical network. These constraints are associated with Kirchhoff's laws for the nodes and transmission lines of the electrical network. We will use the network flow model for the Optimal Power Flow DC as developed in [2, 3, 9].

The mathematical problem of the Optimal Power Flow DC is given by

$$\begin{aligned} \min \quad & \frac{\beta}{2}\left(p^T Q p + c_q^T p\right) + \frac{\alpha}{2} f^T R f \\ \text{s.t } & Af = Hp - d \\ & Tf = 0 \\ & p_{\min} \leq p \leq p_{\max} \\ & f_{\min} \leq f \leq f_{\max}, \end{aligned} \tag{1}$$

where α and β are weights associated to the objective function, $A \in \mathbb{R}^{m\times n}$ corresponds to the incidence matrix of the network, where m is the number of nodes and n is the number of edges in the electrical network. $d \in \mathbb{R}^m$ is the power demand; $f \in \mathbb{R}^n$ represents the active power flow; $p \in \mathbb{R}^g$ represents the active power generation and g is the number of generators in the system; the diagonal matrix $Q \in \mathbb{R}^{g\times g}$ and $c_q \in \mathbb{R}^g$ are associated with the generation cost; the diagonal matrix $R \in \mathbb{R}^{n\times n}$ represents the resistance of the transmission lines; $H \in \mathbb{R}^{m\times g}$ is given by g columns of the identity matrix, and each column represents a generation bus; $T \in \mathbb{R}^{n-m+1\times n}$ is the reactance matrix of the transmission network. $p_{\min}, p_{\max} \in \mathbb{R}^g$ are the power generation's boundaries, and $f_{\min}, f_{\max} \in \mathbb{R}^n$ the power flow's limits. The first two constraints are related to Kirchhoff's laws, and the last ones represent the lower and upper limits of active power generation and active power flow.

The first part of the loss function represents the cost of plant generation, and the second part represents the value of transmission losses.

As mentioned earlier, the energy sector involves uncertain parameters that need to be analyzed stochastically. In this context, we propose to address the uncertainty in demand through a probabilistic approach. To achieve this, it is possible to use a two-stage stochastic programming model with fixed recourse, considering the uncertain demand [1, 5].

Let us consider the generation from hydroelectric plants as first-stage variables, using the variable $p^1 \in \mathbb{R}^{g_1}$, with g_1 hydroelectric plants, while the generation from thermal power plants, denoted by $p^2 \in \mathbb{R}^{g_2}$, involves g_2 thermal power plants. Thus, the stochastic programming problem is

$$\begin{aligned} \min \quad & \frac{\beta}{2}\left((p^1)^T Q_1 p^1 + c_1^T p^1\right) + \mathbb{E}_\epsilon \mathcal{Q}(p^1, \epsilon) \\ \text{s.t. } & p^1_{\min} \leq p^1 \leq p^1_{\max}, \end{aligned} \tag{2}$$

where $\mathbb{E}_\epsilon \mathcal{Q}(p^1, \epsilon)$ is the expected value, $\mathcal{Q}(p^1, \epsilon)$ has variables $p^2 := p^2(\epsilon)$ and $f := f(\epsilon)$ dependent on the random variable ϵ. The probabilistic problem is given by

$$\begin{aligned} \mathcal{Q}(p^1,\epsilon) = \min \quad & \frac{\beta_2}{2}\left((p^2)^T Q_2 p^2 + c_2^T p^2\right) + \frac{\alpha}{2} f^T R f \\ \text{s.t. } \; & H_1 p^1 + H_2 p^2 - Af = d(\epsilon) \\ & Tf = 0 \\ & p^2_{\min} \le p^2 \le p^2_{\max} \\ & f_{\min} \le f \le f_{\max}, \end{aligned} \tag{3}$$

where $H_1 \in \mathbb{R}^{m\times g_1}$, $H_2 \in \mathbb{R}^{m\times g_2}$, $d(\epsilon)$ is the uncertain demand, the diagonal matrix $Q_1 \in \mathbb{R}^{g_1\times g_1}$ and $c_1 \in \mathbb{R}^{g_1}$ are the costs of hydroelectric plants, while $Q_2 \in \mathbb{R}^{g_2\times g_2}$ and $c_2 \in \mathbb{R}^{g_2}$ are the costs of thermal power plants.

Given that there is no closed-form expression for $\mathcal{Q}(p_1,\epsilon)$, then, we adopt an approach of the stochastic problem [6]. In this context, we assume that ϵ is a discrete random variable with N possible scenarios having values $\epsilon_1,\dots,\epsilon_N$, each associated with probabilities $a_1,\dots,a_N$. Consequently, each variable and constraint from the second stage is replicated for each scenario. In other words, we introduce variables p^2_j and f_j associated with scenario $j \in 1,\dots,N$.

The problem (3) is defined for a fixed time. We may extend it with a planning horizon of t hours. Assuming p_h is the target for hydroelectric plants, we obtain the variable p^1_i for stage one, p^2_{ij}, f_{ij} as stage two variables, and d_{ij} as the uncertain demand, where $i \in 1,\dots,t$ and $j \in 1,\dots,N$. Thus, the optimal power flow problem with uncertain demand for a planning horizon of t hours, known as the pre-dispatch problem, is defined as:

$$\begin{aligned} \min \sum_{i=1}^{t} \Bigg\{ & \frac{\beta_1}{2}\left((p^1_i)^T Q_1 p^1_1 + c_1^T p^1_i\right) + \sum_{j=1}^{N} a_{ij}\left[\frac{\beta_2}{2}\left((p^2_{ij})^T Q_2 p^2_{ij} + c_2^T p^2_{ij}\right) + \frac{\alpha}{2} f_{ij}^T R f_{ij}\right]\Bigg\} \\ \text{s.t.} \quad & \sum_{i=1}^{t} p^1_i = p_h \\ & \left.\begin{array}{l} H_1 p^1_i + H_2 p^2_{ij} - A f_{ij} = d_{ij} \\ T f_{ij} = 0 \\ p^1_{\min} \le p^1_i \le p^1_{\max} \\ p^2_{\min} \le p^2_{ij} \le p^2_{\max} \\ f_{\min} \le f_{ij} \le f_{\max} \end{array}\right\} i = 1,\dots,t; j = 1,\dots,N. \end{aligned} \tag{4}$$

Definition 2.1. *The here-and-now solution corresponding to the* ***recourse problem*** *(RP) solution is the optimal solution at the beginning of the planning horizon, that is, the optimal solution of the first stage considering all possible scenarios of uncertain parameters in the second stage. This is* $RP = \min_x \mathbb{E}_\epsilon[z(x,\epsilon)]$.

Since the random variable ϵ is discrete, the (RP) of the pre-dispatch problem is formulated as in equation (4).

Definition 2.2. *The* ***expected solution*** *(EV) is the optimal value considering the mean of the random variable* ϵ.

Definition 2.3. *If* $x = \bar{x}(\bar{\epsilon})$ *the solution of the first stage of EV, the* ***expected value solution*** *(EVS) is the solution of the second stage assuming* x *as an input parameter.*

To efficiently address the pre-dispatch problem under uncertainty, the formulation reveals a block structure that enables scalable solution methods. The next section formalizes this structure to guide the optimization process.

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2.1 Block Structure Formulation

In the present work, we use the variant path-following Interior-Point method for quadratic programming problems [7]. In the process of solving quadratic programming problems with the Interior-Point method, there is an option to employ the Cholesky factorization of the matrix $\tilde{A}\Theta\tilde{A}^T$, where $\tilde{A}$ is the constraint matrix in (4), and Θ is a diagonal matrix with positive entries. However, given the block structure of the matrix $\tilde{A}$, the matrix product $\tilde{A}\Theta\tilde{A}^T$ may have many non-zero entries. To mitigate this, we will rewrite the problem by introducing *splitting variables.*

$$\begin{aligned}
\min \sum_{i=1}^{t}\Bigg\{ & \frac{\beta_1}{2}\left((p_{i1}^1)^T Q_1 p_{i1}^1 + c_1^T p_{i1}^1\right) + \sum_{j=1}^{N} + a_{ij}\left[\frac{\beta_2}{2}\left((p_{ij}^2)^T Q_2 p_{ij}^2 + c_2^T p_{ij}^2\right) + \frac{\alpha}{2} f_{ij}^T R f_{ij}\right]\Bigg\} \\
\text{s.t} \quad & \sum_{i=1}^{t} p_{i1}^1 = p_h \\
& \left.\begin{array}{l} H_1 p_{ij}^1 + H_2 p_{ij}^2 - A f_{ij} = d_{ij} \\ T f_{ij} = 0 \\ p_{\min}^2 \le p_{ij}^2 \le p_{\max}^2 \\ f_{\min} \le f_{ij} \le f_{\max} \\ p_{\min}^1 \le p_{ij}^1 \le p_{\max}^1 \end{array}\right\} i = 1,\dots,t; j = 1,\dots,N-1, \qquad (5)\\
& p_{ij}^1 - p_{i,j+1}^1 = 0, \qquad j = 1,\dots N-1, i = 1,\dots,t. \qquad (6)
\end{aligned}$$

The constraints (6) are the splitting variables. To check the block structure of optimization problem (5)-(6), we define the following notation

$$\tilde{A} = \begin{bmatrix} \tilde{I} & & & & \\ M_1 & W_1 & & & \\ M_2 & & W_2 & & \\ \vdots & & & \ddots & \\ M_N & & & & W_N \end{bmatrix}, \qquad (7)$$

where

$$\tilde{I} = [I^1 \dots I^t], \qquad M_j = \begin{bmatrix} \tilde{H}_1 & & \\ & \ddots & \\ & & \tilde{H}_t \end{bmatrix}, \; W_j = \begin{bmatrix} \tilde{T}_1 & & \\ & \ddots & \\ & & \tilde{T}_t \end{bmatrix}, \; j = 1,\dots,N, \qquad (8)$$

for $I^i = I_{g_1}$, with $i = 1,\dots,t$, where I_{g_1} is an identity matrix $g_1 \times g_1$, and

$$\tilde{H}_i = \begin{bmatrix} H_1 \\ 0 \end{bmatrix}, \; \tilde{T}_i = \begin{bmatrix} H_2 & -A \\ 0 & T \end{bmatrix}, \qquad i = 1,\dots,t. \qquad (9)$$

Therefore, using the matrix notation defined in (8) and (9), the constraint matrix of the problem (5)-(6) without the lower and upper bounds is given by

$$\tilde{A} = \begin{bmatrix} \tilde{I} & & & & & & & & & & \\ M_1 & W_1 & & & & & & & & & \\ & & M_2 & W_2 & & & & & & & \\ & & & & M_3 & W_3 & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & M_{N-1} & W_{N-1} & & \\ & & & & & & & & & M_N & W_N \\ I & & -I & & & & & & & & \\ & & I & & -I & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & & I & & -I & \end{bmatrix}. \tag{10}$$

The splitting variables guarantee that the first-stage decision variables remain feasible under every scenario, producing the block-diagonal structure shown in matrix (10). Because the same first-stage decisions are applied across all scenarios, the non-anticipativity requirement is satisfied. This block-structure methodology was studied by [4] and later extended to two-stage stochastic problems in [6] and multistage stochastic optimization in [5].

3 Computational Tests

This section presents the results of our computational study. The problems were performed in a Linux Environment, on an Intel Core i7-3770K, processor 3.50GHz with 32 GB RAM, using the specialized package for Multistage Stochastic Programming MSSO-BlockIP [5]. To analyze the computational results, we introduce the following definitions from [1].

3.1 Network Energy

We tested the IEEE30 bus system model. In numerical tests, we defined 2/3 of the generation capacity corresponding to hydroelectric plants and the remaining 1/3 capacity to thermal power plants. The lower limit of the generators was set to zero for both hydroelectric and thermal power plants. The upper limit of active power flow was defined according to the system IEEE30. The costs of active power generation for thermal power plants were considered fifteen times the costs of generation for hydroelectric plants.

3.2 Energy Demand

To analyze the optimization of the *RP* through a block-based method, we use data from Brazil for demand. The hourly load curve dataset from the *Operador Nacional de Sistema Elétrico* (ONS) [10] serves as the basis for this research. The data were collected for each hour from 2009 to 2019.

3.3 Scenario Definition

We generate 10 scenarios for the demand over a 24-hour period. Eight scenarios are derived from historical data, each with a probability of 12%. We employ a Monte Carlo simulation-based sampling approach to generate these scenarios. Additionally, scenarios 9 and 10, each with a 2%

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probability, account for a 20% increase/decrease relative to the mean data presented in Section 3.2.

For the present work, we obtain RP solution according to the block structure of the constraint matrix defined in (10) using the MSSO-BlockIP package [5].

In Table 1, we analyze the obtained results for planning 24 hours by modifying the values of α and β defined in equation (5)-(6). To minimize generation costs, we assume $\alpha = 0$ and $\beta_1 = \beta_2 = 1$; and for transmission costs, we set $\beta_1 = \beta_2 = 0$ and $\alpha = 1$. Additionally, the optimization problem for both generation and transmission is tested assuming $\alpha = \beta_1 = \beta_2 = 1$.

Table 1: IEEE30 values for transmission and/or generation optimization approach.

Optimization	Value of RP
Generation	$7.8 \cdot 10^5$
Transmission	$9.6 \cdot 10^5$
Generation and Transmission	$1.5 \cdot 10^6$

Table 2 compares the RP problem solved with variable-splitting via MSSO-BlockIP against the same problem solved with Gurobi-Python (UNICAMP academic license). The MSSO-BlockIP approach shows clear gains, reducing both the total running time and the number of iterations relative to Gurobi-Python.

Figure 1 presents the optimal value for the Generation and Transmission case, considering the respective values of α and β as specified. The graph illustrates the satisfaction of the requirement $EVS \geq RP$ (see [1]). Additionally, the hours of highest cost occur in the afternoon, while the lowest-cost hours are observed late at night.

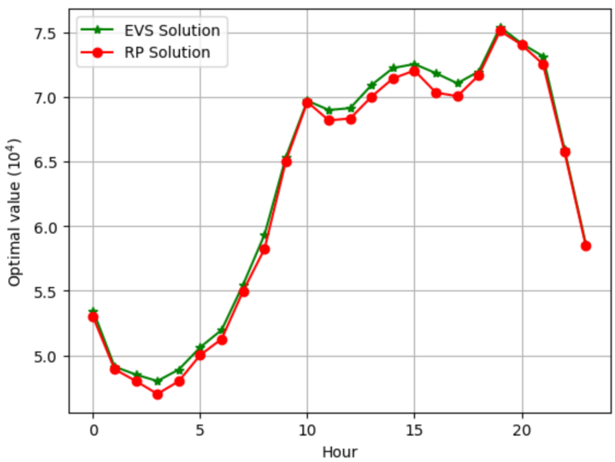

Figure 1: Optimized cost per hour. Font: author.

Table 2: IEEE30 number of iterations for comparing splitting variables approach for Generation and Transmission problem.

Optimization	Number of iterations	Time(s)
RP MSSO-BlockIP	11	0.14
RP Gurobi-Python	13	0.20

4 Conclusions

We formulate the optimal power flow problem over a t-hour planning horizon considering demand uncertainty. Using a two-stage stochastic programming model, the first stage covers hydroelectric generation and the second thermal generation. Demand uncertainty is modeled as a discrete random variable, generating multiple scenarios. We apply a block strategy with splitting variables to create a block-structured constraint matrix, enabling the use of specialized optimization methods. Computational tests show improvements in iterations and runtime compared to general-purpose software, demonstrating the effectiveness of our approach.

This work contributes to new solution methods for various energy system optimization problems, including maintenance and security constraints.

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