Topological Derivative-Based Shape Optimization with Pointwise Temperature Constraints and Applications

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The use of shape and topology optimization to design efficient thermal management devices has become a subject of significant research interest. Early studies primarily focused on minimizing thermal compliance or maximizing heat dissipation, often relying on conduction-based models [3]. To better align with real-world applications, more recent advancements have incorporated convection and radiation effects, as well as multi-physics considerations, such as fluid-thermal and thermo-mechanical coupling. Another important aspect of such applications is the need to impose temperature constraints to ensure that heat-generating devices operate within a specific temperature range, preventing component overheating or undesirable phase transitions. In this work we address the issue of realizing pointwise temperature constraints through a topological derivative-based strategy.

Given a fixed reference domain $\mathcal{D} \subset \mathbb{R}^2$ and a set $\mathcal{O}$ of admissible subdomains of $\mathcal{D}$, our goal is to solve the following optimization problem

$$\min_{\Omega \in \mathcal{O}} \quad \mathcal{J}(\Omega) \quad \text{subject to} \quad T(x) \leq T^*(x), \quad \forall x \in \mathcal{D}, \tag{1}$$

where T is the solution of the steady-state heat conduction equation posed on the optimization domain, $\mathcal{J}$ is a cost functional and T^* is a given limit temperature. The material properties of the domain are defined by piecewise constant functions, in a way that Ω can be understood as a stiff and highly conductive material, while $\mathcal{D}\backslash\Omega$ is modeled as a very compliant and poor conductor material. We consider two distinct applications. First, we investigate the design of optimal heat sink devices. The domain is subjected to only thermal loads and the cost functional is defined as a combination of the volume of the highly conductive phase and the thermal compliance. Next, motivated by applications to electric vehicles and aircrafts, as described in [4, 7], we consider the design of an aluminum battery pack. Additional mechanical loads are introduced, so that the thermal problem is coupled with the thermo-elasticity partial differential equation. The cost functional is defined as a linear combination of the total energy of the system and the volume of the aluminum structure.

We approximate the pointwise constraint $T(x) \leq T^*(x)$ using the following quadratic penalty functional

$$\mathcal{G}(\Omega) := \int_{\mathcal{D}} (T - T^*)_+^2 \, dx, \tag{2}$$

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where $(T-T^*)_+ := \max\{T-T^*, 0\}$. Thus, the original optimization problem can be restated with a single equality constraint. Then, in order to apply the topological derivative method, we compute the first-order asymptotic expansion of $\mathcal{G}$ with respect to an infinitesimal singular perturbation of the geometry [6]. While the topological derivative of energy-type functionals are well established in the literature, our main contribution with this work is the discussion on the existence of the topological derivative of (2). This discussion was carried out in [2].

The topology optimization problem is then tackled with the algorithm proposed by Amstutz and Andrä [1], which combines a level-set representation of the domain with the use of the topological derivative of the cost functional as a steepest-descent direction. The numerical results confirm the expected trade-off between the optimal domain's volume and the imposed temperature constraints, demonstrating the necessity of allocating more conductive material as stricter constraints are enforced. Furthermore, they reveal a tendency to concentrate material along the shortest paths between the hottest regions and heat sink areas.

It is worth mentioning that, although we are motivated by thermal applications, problem (1) represents a more general class of state-constrained problems in shape and topology optimization. Therefore, our approach could be extended to address problems with different physical motivations. Furthermore, this type of constraint can also be used to impose connectivity constraints for manufacturing process, which is a current and relevant application of the present study [5].

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