Principled and Modern Bayesian Analysis with the Normalised Power Prior

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Incorporating historical information is often essential for gaining deeper insights into new clinical trials. Regulatory agencies, for example, may consider historical trial data to accelerate the approval process of new medical research. In this context, the elicitation of informative priors is primordial for historical borrowing based on data compatibility. A widely used approach for this purpose is the *power prior* [4–6], which introduces a power parameter η, usually between 0 and 1, to control the influence of the historical data likelihood.

To illustrate the challenges with this approach, consider the historical data D_0 and its likelihood $L(D_0 \mid \theta)$, where $\theta \in \Theta \subset \mathbb{R}^q$ represents the parameter of interest. The original formulation of the power prior is given by:

$$\pi(\theta \mid D_0, \eta) \propto L(D_0 \mid \theta)^\eta \pi_0(\theta), \tag{1}$$

where $\pi_0(\theta)$ is the initial prior of θ. Extending this framework, we may also assign an initial prior $\pi_A(\eta)$ to the power parameter, leading to the joint prior:

$$\pi(\theta, \eta \mid D_0) \propto L(D_0 \mid \theta)^\eta \pi_0(\theta)\pi_A(\eta)c^{-1}(\eta), \tag{2}$$

where $c(\eta) = \int_\Theta L(D_0 \mid \theta)^\eta \pi_0(\theta) d\theta$.

Two main challenges arise with this approach. The first question is how to choose the prior on the discounting parameter that incorporates historical information optimally into the posterior distribution. Second, the computation of the posterior distribution presents difficulties, particularly due to the intractability of the normalizing constant [1], which complicates efficient sampling methods. This, combined with the need to recompute these quantities when carrying out simulations, leads to difficulties in producing a good Bayesian analysis.

We propose a modern approach to Bayesian inference under the power prior, starting with thorough *prior predictive simulations* [3] to explore the pushforward distribution of observables under the chosen prior – characterised here by either a fixed η or its prior distribution, π_A. Moreover, we advance the idea of employing *posterior predictive checks* [7] in order to gauge prior-data conflict and understand the overall fit of the model. Finally, we propose to explore the idea of **prior effective sample size** (PESS) [2] as a tool to quantify the amount of information introduced into the analysis through the NPP.

References

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