

Quantum Ray Tracing

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Since Feynman's influential work [2], quantum computing has developed into a robust computational paradigm, offering new approaches to problems that challenge the limits of classical systems. This potential is exemplified by two cornerstone algorithms. Shor's algorithm [7] revolutionizes integer factorization, reducing its complexity from sub-exponential, as achieved by the best classical methods, to polynomial time. Similarly, Grover's algorithm [3] offers a quadratic speedup for searching unstructured databases, decreasing the complexity from $O(N)$ to $O(\sqrt{N})$. This efficiency gain is achieved through the amplification of amplitudes for "marked" states, making them more likely to be measured. Grover's algorithm is particularly versatile, enabling the reformulation of numerous problems as search tasks, thereby broadening its scope of application across various fields.

Quantum computing operates on principles of quantum mechanics, utilizing quantum bits (qubits) as the fundamental units of information [5]. A qubit is described by a superposition state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad (1)$$

where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$. Quantum operations are represented by unitary matrices acting on these states, and measurement collapses the qubit to one of the basis states $|0\rangle$ or $|1\rangle$, with probabilities $|\alpha|^2$ and $|\beta|^2$, respectively. The tensor product of individual qubit states describes multi-qubit systems. For n qubits, the state is represented as:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle, \quad (2)$$

which lies in a 2^n -dimensional Hilbert space. This high-dimensional space enables quantum oracles to act on superpositions of many states simultaneously. In Grover's algorithm, the oracle uses this property to evaluate multiple solutions in parallel, using a combination of phase inversion and diffusion operations to amplify the probability of measuring desired solutions, achieving a quadratic speedup over classical search methods.

One application of Grover's algorithm lies in ray tracing [8], a widely used technique in computer graphics to compute intersections of rays with objects in three-dimensional scenes. Classical ray tracing has $O(N)$ complexity concerning the number of geometric primitives in the scene, where N represents the primitives to be tested for intersection. By integrating Grover's algorithm with quantum minimization methods [1], it is possible to achieve a quadratic speedup, reducing the complexity to $O(\sqrt{N})$. This improvement significantly impacts performance for large-scale geometric models.

The objective of this work is to implement a quantum ray tracing solution using Grover's algorithm, inspired by [6]. This involves constructing quantum circuits for oracles, describing their mathematical structure rigorously, and demonstrating the feasibility of the approach through a proof of concept. Key components involve defining matrix representations of quantum operators,

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developing appropriate circuits to compute intersections, and designing the oracles necessary for implementing the quantum minimum finding algorithm.

The implementation is designed using the Qiskit framework [4], ensuring accessibility and reproducibility. The source code will be made publicly available to encourage extensions and adaptations for future research in quantum computing and computer graphics.

Acknowledgments

This study was partially funded by FAPEMIG and the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

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