Nash Equilibria in Transmitter-Jamming Games for Fleets of UAVs

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Abstract. We consider a scenario where two fleets of unmanned aerial vehicles are tasked with communicating information among fleet members while simultaneously trying to prevent the opposite fleet from achieving their goal. Using the properties of the electromagnetic spectrum, we model the task of each fleet as a continuous optimization problem, leading to a highly nonlinear zero-sum Nash game encoding the preferences of both fleets. A fixed-point reformulation of the game provides us with a block-descent algorithm, for which we provide some first numerical results.
Keywords. Nash games, UAVs, Drones, Electronic Warfare, Fixed-Point Algorithms

1 Introduction

Electronic Warfare (EW) is a military domain that involves the use of the electromagnetic spectrum to gain an advantage in various operational environments. An electronic attack like **jamming** aims to interfere with the adversary's ability to collect or disseminate information. Communications jamming is directed at disrupting radio systems & involves transmitting interfering signals on the same frequency as the targeted communication system, making it difficult for the intended message to be received. Unmanned aerial vehicles (UAVs or drones) and EW represent critical components in modern military and security strategies. In the context of EW, drones play a significant role both as platforms for EW systems and as potential targets. Drones equipped with EW capabilities can stage jamming attacks, whilst being susceptible to EW themselves.

In this work, we consider two fleets of UAVs that are antagonistic to each other. Members or agents of each fleet are tasked with sharing information between other members of their fleet, i. e. they want to send and receive data. However, they also have the option to degrade or interrupt the information flow between members of the other fleet. Of crucial importance in this model is the ability of each agent to be mobile, i. e. to spend some of its energy to move around in space. By moving closer to an agent of the same fleet, information exchange between these two agents is improved. By moving closer to agents of the opposite fleet, interrupting the information flow of the opposition (jamming) is ameliorated.

We model this problem as a novel Nash game, where both players control their relevant fleets. For each agent, decision variables include the power expenditure to communicate with other agents of the same fleet, power expenditure to jam communication between pairs of agents in the opposite fleet, and power expenditure for and direction of any movement. Previous attempts at modeling this problem either use stationary agents only [8] or allow only one jamming agent to move [2]. Our communication and jamming model reflects the underlying physical reality of communication within the electromagnetic spectrum and contains no linearizations. In this note, we focus on the

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electromagnetic and spatial aspects of the problem and leave considerations of flight dynamics to a more extensive version of this work.

Our model follows the work of Khanafer et al [8], who considered a similar Nash game with two fleets of stationary agents, each fleet consisting of only two agents. Gupta et al [7] consider a game with incomplete information in which one player tries to jam the connection between a transmitter and a receiver. Other game-theoretic models on jamming UAVs are discussed in [4, 9, 10, 14, 15, 17], while Mkiramweni et al [11] provide a survey of game-theoretic approaches for UAVs and wireless networks.

2 The Model

Suppose we have given two finite index sets F and G, with $F \cap G = \emptyset$, denoting two fleets of agents or UAVs. We use indices $i, j, k, \ell \in F \cup G$ to indicate agents, and will usually use the convention $i, j \in F$ and $k, \ell \in G$, unless explicitly stated otherwise. For each $i \in F$, $k \in G$, we have been given locations $x_i^0, x_k^0 \in \mathbb{R}^2$ and a maximum amount of energy to spend, P_i^{max}, P_k^{max}.

Suppose that agents i and j are on the same team and that i wants to communicate some data to j. Following [8], if i uses $p_{i,j}$ amount of power for this communication (i. e. it sends a signal of strength $p_{i,j}$), then j receives an amount of power $p_{i,j}^R$ equal to

$$p_{i,j}^R = \varrho p_{i,j} d_{i,j}^{-\alpha} \tag{1}$$

with $d_{i,j} = \|x_i - x_j\|$ the Euclidean distance between i and j. In the above and in what follows, we always assume $x_i \neq x_j$, and in all formulations of optimization problems, we consider implicit constraints to this effect. The constant α is the so-called path-loss coefficient that depends on the environment (atmospheric conditions, terrain, etc) in which i and j try to communicate and is usually between 2 and 4. The constant ϱ depends on the design of the antennas of i and j and the signal wavelength in use; in what follows we will assume that ϱ does not depend on i or j.

A crucial quantity is now the signal-to-interference-and-noise ratio $\mathrm{SINR}_{i,j}$, defined as

$$\mathrm{SINR}_{i,j} = \frac{p_{i,j}^R}{\sigma^2 + I_j}, \qquad (i, j \in F, i \neq j). \tag{2}$$

Here, $\sigma^2 > 0$ represents background noise due to solar radiation etc. Furthermore, I_j is the total interference power received by j due to jamming. Suppose that each agent k from the opposite team uses a power of $p_{k,j}^J$ to jam all communications of player j. Then we have

$$I_j = \sum_{k \notin F} \varrho p_{k,j}^J d_{k,j}^{-\alpha}, \qquad (j \in F) \tag{3}$$

and the summation is over all agents of team G. In total, we have

$$\mathrm{SINR}_{i,j} = \frac{\varrho p_{i,j} d_{i,j}^{-\alpha}}{\sigma^2 + \varrho \sum_{k \notin F} p_{k,j}^J d_{k,j}^{-\alpha}}, \qquad (i, j \in F, , i \neq j). \tag{4}$$

It is this quantity SINR that agents i and j want to maximise, as it is closely related to various performance indices used in wireless communications. For digital data, the bit error rate (the average number of bits received by agent j which are received incorrectly) can be directly expressed in terms of $\mathrm{SINR}_{i,j}$, see [12].

In total, the **first** objective of the fleet of agents F is thus

$$\max_{p_{i,j}, x_i, x_j} \sum_{\substack{i,j \in F \\ i \neq j}} \frac{\varrho p_{i,j} d_{i,j}^{-\alpha}}{\sigma^2 + \varrho \sum_{k \notin F} p_{k,j}^J d_{k,j}^{-\alpha}}, \tag{5}$$

where the decision variables are the power levels $p_{i,j}$ $(i, j \in F,\, i \neq j)$ and the locations $x_i, x_j \in \mathbb{R}^2$ $(i, j \in F)$ determining $d_{i,j} = \|x_i - x_j\|$, $d_{k,j} = \|x_k - x_j\|$ $(k \notin F)$. Note that the power levels $p^J_{k,j}$ are decision variables of the opposite fleet with agents indexed by k. In turn, fleet F controls decision variables $p^J_{j,k}$ $(j \in F, k \notin F)$ that show up in the corresponding objective of the opposite fleet.

However, note that in this objective the effect of jamming agents of fleet G is not taken into account. We therefore consider now the first objective of fleet G, that fleet G wants to maximize, as an objective to be minimized. The **second** objective of fleet F is thus

$$\min_{p^J_{i,\ell}, x_i} \sum_{\substack{k,\ell \notin F \\ k \neq \ell}} \frac{\varrho p_{k,\ell} d_{k,\ell}^{-\alpha}}{\sigma^2 + \varrho \sum_{i \in F} p^J_{i,\ell} d_{i,\ell}^{-\alpha}} \tag{6}$$

Each agent i has a starting position $x_i^0 \in \mathbb{R}^2$. If the agent then moves to x_i before sending or jamming, then this accrues an energy expenditure of $c_i \|x_i - x_i^0\|$, where c_i is a suitable constant. For fleet F, we have thus decision variables as follows: $p_{i,j}$ $(i, j \in F,\, i \neq j)$ power used to communicate from i to j; $p^J_{i,k}$ $(i \in F,\, k \notin F)$ power used to jam the communication of opposite agent $k \notin F$; x_i $(i \in F)$ location of agent i after movement and before communication and jamming takes place. For ease of notation, we collect all these decision variables into a vector x. Fleet G has exactly the same types of decisions to make for members of fleet G, and we denote by y the corresponding vector of decision variables.

We can then write the two objectives of fleet F as one bi-objective optimization problem

$$\max_{x=(p_{i,j}, p^J_{i,k}, x_i)} \begin{bmatrix} f_1(x,y) \\ -f_2(x,y) \end{bmatrix} := \begin{bmatrix} \displaystyle\sum_{\substack{i,j \in F \\ i \neq j}} \frac{\varrho p_{i,j} \|x_i - x_j\|^{-\alpha}}{\sigma^2 + \varrho \sum_{k \notin F} p^J_{k,j} \|x_k - x_j\|^{-\alpha}} \\ \displaystyle -\sum_{\substack{k,\ell \notin F \\ k \neq \ell}} \frac{\varrho p_{k,\ell} \|x_k - x_\ell\|^{-\alpha}}{\sigma^2 + \varrho \sum_{i \in F} p^J_{i,\ell} \|x_i - x_\ell\|^{-\alpha}} \end{bmatrix}. \tag{7}$$

Note that this problem has parameters y, which are the decision variables of the opposite fleet.

As constraints, we have the **power constraint** for each agent i,

$$\sum_{\substack{j \in F \\ j \neq i}} p_{i,j} + \sum_{k \notin F} p^J_{i,k} + c_i \|x_i - x_i^0\| \leq P_i^{max} \qquad (i \in F), \tag{8}$$

a constraint on the movement of each agent i,

$$\|x_i - x_i^0\| \leq \delta_i \qquad (i \in F) \tag{9}$$

with $\delta_i > 0$ some given parameter, and $p_{i,j}, p^J_{i,k} \geq 0$ $(i \in F, j \in F, i \neq j, k \notin F)$. We denote the feasible set of fleet F by C_1. The constraints for fleet G have exactly the same structure, and we denote the corresponding feasible set by C_2. With this, fleet G faces the problem

$$\max_{y \in C_2} \begin{bmatrix} f_2(x,y) \\ -f_1(x,y) \end{bmatrix}, \tag{10}$$

parameterized in x. The decision variables of fleet F.

Remark 2.1. *Note that it is straightforward to generalise the problem above to one in which agents can only communicate with selected members of their team.*

Remark 2.2. *Without loss of generality, we can assume that the power constraint (8) is active, as the first objective is strictly monotone in the $p_{i,j}$ and the second objective is strictly monotone in the $p^J_{i,\ell}$.*

In what follows we **scalarize** both problems in the standard fashion; the two fleets then face the problems

$$\max_{x\in C_1} f_1(x,y) - f_2(x,y) \tag{11}$$

$$\max_{y\in C_2} f_2(x,y) - f_1(x,y). \tag{12}$$

Thus, the objectives of the two fleets are essentially the same; they just 'swap' the variables x and y around. The proposition below follows directly from this observation and formalizes it.

Proposition 2.1. *Let both fleets have the same number of agents. Suppose that all parameters c_i are identical to each other and that all parameters P_i^{max} are identical to each other. Assume further that the set of starting points of the two fleets are identical, $\{x_i^0 \mid i \in F\} = \{x_k^0 \mid k \notin F\}$. Then the two sets of feasible points are identical, $C_1 = C_2$. Moreover, we have*

$$f_1(x,y) = f_2(y,x). \tag{13}$$

Considering the problem at hand as a **zero-sum Nash game**, we are looking for an equilibrium, i. e. a pair of decisions $(x^*, y^*) \in C_1 \times C_2$ for which we have

$$f_1(x^*,y^*) - f_2(x^*,y^*) \quad\geq\quad f_1(x,y^*) - f_2(x,y^*) \qquad \forall x \in C_1, \tag{14}$$

$$f_2(x^*,y^*) - f_1(x^*,y^*) \quad\geq\quad f_2(x^*,y) - f_1(x^*,y) \qquad \forall y \in C_2, \tag{15}$$

Since all functions involved are sufficiently smooth, we can characterise such equilibria by their KKT conditions. To solve such conditions numerically, one often resorts to an MPEC (Mathematical Program with Equilibrium Constraint) solver. However, extensive numerical tests with state-of-the-art MPEC solvers have shown that this approach does not seem to be numerically feasible in our case, as all such solvers get stuck at infeasible points. In the next section, we will therefore provide an alternative numerical approach, based on a fixed-point formulation of the problem.

3 Fixed-Point Formulations

Denote by θ_F and θ_G the objectives (to be minimized) of both fleets F and G, i. e. $\theta_F = -f_1 + f_2$ and $\theta_G = -\theta_F$. For a given $(\hat{x}, \hat{y})$ we consider the optimization problem

$$\min_{x,y} \qquad \theta_F(x,\hat{y}) + \theta_G(\hat{x},y) \tag{16}$$

$$\text{subject to} \qquad x \in C_1, y \in C_2. \tag{17}$$

Suppose that (x^*, y^*) is a solution to this problem. It is well known (see [13]) that (x^*, y^*) is a Nash equilibrium if $(x^*, y^*) = (\hat{x}, \hat{y})$, i. e. if (x^*, y^*) is a fixed point of the solution mapping of (16)–(17).

A standard approach to circumvent numerical issues related to the nonuniqueness of any optimum of (16) is to add a regularisation term and consider

$$\min_{x,y} \qquad \theta_F(x,\hat{y}) + \theta_G(\hat{x},y) + r(\|x-\hat{x}\|^2 + \|y-\hat{y}\|^2) \tag{18}$$

$$\text{subject to} \qquad x \in C_1, y \in C_2 \tag{19}$$

with a parameter $r \geq 0$.

The following block-descent algorithm then provides a fixed-point iteration for our Nash game:

1. Choose $r \geq 0$ and $\hat{x} \in C_1$ and $\hat{y} \in C_2$.

2. Solve (18)–(19). Denote the result by (x, y).
3. Set $(\hat{x}, \hat{y}) := (x, y)$.
4. Update r.
5. Goto step 2.

Step 2 of the algorithm can obviously be split up into solving the two problems

$$\min_{x} \quad \theta_F(x, \hat{y}) + r\|x - \hat{x}\|^2, \tag{20}$$

$$\min_{y} \quad \theta_G(\hat{x}, y) + r\|y - \hat{y}\|^2, \tag{21}$$

which can be done in parallel. Both correspond to "best response" optimization problems, i. e. (20) computes the strategy of fleet F if it is known that fleet G will deploy strategy $\hat{y}$. The overall algorithm is thus one of successive improvements in strategies x and y. A convergence proof for the case of the block-descent algorithm applied to convex functions can be found in [5], who also describe various adaptations like solving only for ε-minima in each step, appropriate strategies for changing the parameter r, etc. Further convergence results are discussed in [1, 6, 16] under quite general conditions.

Theorem 3.1. *Denote by $Z := (z_k)_k = (x_k, y_k)_k$ the sequence generated by the above algorithm. Let the sequence of regularization parameters $(r_k)_k$ be such that $0 < r_L \leq r_k \leq r_U$ for some constants $r_L, r_U > 0$.*

1. *Suppose that Z has an accumulation point. Then every accumulation point of Z is a critical point to problem (16)–(17).*
2. *Denote by Ξ the set of critical points of our problem. Then $dist(z_k, \Xi) \to 0$. If Ξ contains uniformly isolated points, then Z converges to a point in Ξ.*
3. *Let α be an even integer and let (x_1, y_1) be sufficiently close to a global minimizer. Then Z converges to a global minimizer.*

Proof.

1. This follows directly from Proposition 7 in [6] or from Theorem 2.3 of [16].
2. The sequence Z is bounded, as it contains only feasible points. Moreover, the set of feasible points C_1 does not depend on the decisions y of the second player, fleet G, and vice versa. The result then follows from Corollary 2.4 in [16].
3. Let us rewrite problem (16)–(17) as

$$\min_{z=(x,y)} f(z) + r_1(x) + r_2(y), \tag{22}$$

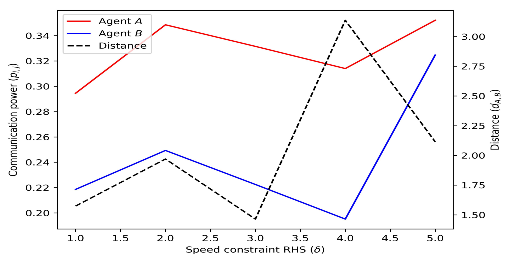
where f is the original objective and r_i is the indicator function of C_i. Both sets C_i are semi-algebraic. Likewise, it is easy to see that the function f is semi-algebraic for even integers α. Therefore, the unconstrained problem (22) has a semi-algebraic objective which thus fulfils the Kurdyka-Łojasiewicz inequality at every global solution. Next, we note that f is Lipschitz-continuous on $C_1 \times C_2$, and that the proof of Lemma 2.6 of [16] only requires this Lipschitz-continuity on the set of feasible points. Thus, all assumptions for a slightly strengthened version of Lemma 2.6 of [16] are fulfilled, and the result follows with Corollary 2.7 of [16].

4 A Numerical Example

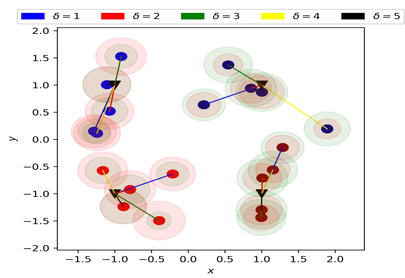
In this section, we provide an illustrative example to demonstrate the efficacy of our approach. We consider two fleets of agents $F = \{A, B\}$ and $G = \{\alpha, \beta\}$. For all UAVs i we use the same parameters $c_i = 0.2$, $P_i^{max} = 1$, as well as the path loss coefficient $\alpha = 1$ and the constants $\varrho = 1$, $\sigma^2 = 1$. Furthermore, we consider starting positions $x_A^0 = (-0.1, 1), x_B^0 = (0.1, 1), x_\alpha^0 = (-1, -1)$, and $x_\beta^0 = (1, -1)$. The regularization parameter of the algorithm is set to $r = 10^{-3}$.

We have implemented the block-descent method and both problems (20) and (21) in AMPL release 20230430 and solved them with KNITRO v13.1 [3]. Numerical results exploring various parameter sets for both problems indicate that KNITRO always solves our problems to optimality in less than 15 iterations. Likewise, for the block-descent algorithm the objective function value $\theta_F + \theta_G$ always converges within 5 steps. This behaviour is in line with Theorem 3.1.

To provide some insight into the dynamics of the scenario, we now vary the parameter $\delta := \delta_i$ for all agents $i \in F \cup G$ and consider optimal solutions for the five Nash games specified by the parameters above and $\delta = 1, \dots, 5$. Figure 1 (a) shows $p_{A,B}$, $p_{B,A}$, and $\|x_A - x_B\|$ varying over δ. Figure 1 (b) shows the optimal locations of all four agents for the five games considered. It can be seen that both the spatial characteristics of the equilibria computed as well as their power expenditures vary considerably, indicating the importance of considering as decision variables not only power expenditure but also the location of UAVs in games that model electronic warfare.



(a) Power expenditure for agents of team F and distance between agents at the computed equilibria, for varying values of δ.



(b) Locations of agents at different equilibria. The radii of the circles are proportional to the power spent. The colours green and red represent communications and jamming, respectively.

Figure 1: Numerical results for different values of δ. (Source: from the authors.)

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