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# Preservation of Structures Through a TLCD in the U-Shaped Reservoir

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**Abstract**. A control system can significantly improve the strength of structures to mitigate vibrations or their effects on the structure, keeping them within limits safe and meeting safety standards. One of the ways to promote this dampening of oscillations on the structure to be protected is the tuned liquid column damper (TLCD). This system promotes the damping of the structure's oscillations through the oscillation of the liquid in a U-shaped reservoir, in which the liquid moves in opposition to the structure's movement. The damping force is associated with the energy dissipation when the liquid passes through the orifice in its horizontal part. We take this non-conservative force with a linear dependence on the velocity. With this simplified model, we obtained analytical expressions for the description of the damping that is within the expected.

**Keywords**. Vibration, Damping, Oscillation, TLCD, Energy Dissipation.

## 1 Introduction

There is no doubt of the importance of studying oscillatory or vibrating systems. Oscillations are present in many situations in nature, such as the movement of the eardrums that allows us to hear, the speech that is associated with the movement of the larynx and the tongue, airplane turbines and wings, and a large number of mechanical systems [5], buildings, stability in electrical systems [1], temperature in reactions [2], harmonic oscillations in quantum systems [6, 9] and other with great practical application.

In this work, a model deals with a real problem of great application in civil and mechanical engineering related to the preservation of structures when submitted to earthquakes. We built a model that couples a structure, and beneath this is placed a U-shaped tube artifact partly filled with a liquid. In this case, the tube will serve as the structure damper. This system is called a tuned liquid column damper (TLCD) [8, 10] and belongs to a class of passive control devices that is an option used to mitigate structural vibrations. The movement differential equations are obtained through the Lagrangian formalism, and the equation system is solved using the Laplace transform.

## 2 Damping by U-tube system

When a body is submitted to external stimulation, an oscillation may be generated, causing the body to be exposed to a level of vibration corresponding to the force it has been subjected to.

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Mechanisms of vibration absorption are necessary to reduce vibration levels in several engineering applications, such as bridges, buildings, and wind turbines.

Dynamic absorbers are oscillators attached to the structure and adequately tuned to a frequency close to a vibration mode or a harmonic excitation; the kinetic power transfer occurs from the structure to be preserved to the absorber.

There are several studies concerning the various types of absorbers. One of the most promising energy absorption devices is the absorbers that use liquid due to mainly, its easy implementation, low maintenance requirements, and low cost, which represent some of the advantages of this type of system, which is usually rectangular or quadrangular with water inside at atmospheric pressure and placed on the highest floor of the building, where the significant displacements occur.

In particular, TLCD [3, 4, 7, 11] belongs to a class of passive control that uses liquids in a U-shaped container to control the vibration of the structure to be preserved (primary structure) [8]. This mechanism of vibration control has been studied in various types of structures, such as, for instance, the control of vibration in tall buildings and in wind turbines that are often shaken, adding to the fact that in the last decades, there has been a trend towards the construction of buildings increasingly higher using alternative material resources to reduce cost, giving rise to lighter and more flexible structures, subjected to low damping that requires more and more a system that protects the power of winds, to resist dynamic actions to which they are subject. An important aspect to consider while building a structure that lacks a damping system is the vulnerability caused by this absence, leading to material and economic losses and human losses [7].

This work will address the TLCD. These systems allow a dampening of the structure through the oscillation of the liquid column in its recipient tube shaped in U. They are viable for construction and, based on the inertia of the liquid, oppose the movement of the structure.

The idea behind this mechanism, TLCD, is to decrease the structure's oscillation amplitude through oscillations out of phase with the auxiliary system, minimizing the structure's movement and dampening the quakes.

Water will be used inside the tube for the model proposed in this work, illustrated in Figure 1. The model seeks to simulate shakes made to a building, such as an earthquake hitting a building. In this prototype, a structure is represented by a box (1) in which a U-shaped tube (2) is fixed with a fluid inside it. The building is connected to a vertical wall (3) by a spring (4). The spring simulates the movement transferred from the ground to the building.

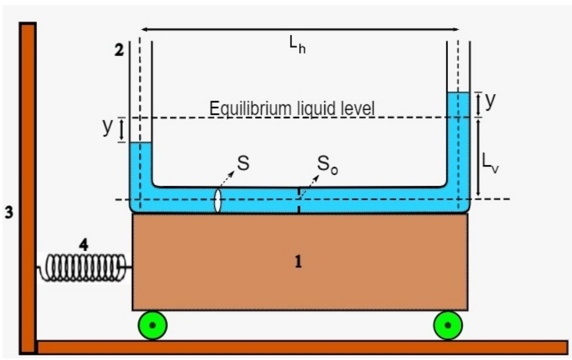


Figure 1: Experimental proposal for the use of the TLCD. Source: by the authors.

For the maximum effectiveness of the damping system, the frequency of fluid movement must be in tune with the structure's natural frequency. In addition, the damping ratio of the structure's motion must be adjusted to an optimal value. For this type of response, in addition to choosing the fluid, one should seek the proper scaling of parameters $L_v, L_h, S$, and $S_0$.

We will describe the movement of the fluid inside the U-tube container with the following characteristics: 1) the flow is stationary; that is, the speed of the moving fluid at a given point does not vary over time. 2) the flow is incompressible; the fluid density does not vary, independent of the position and the time. 3) the flow is irrotational; that is, no element of the fluid in motion spins around its axis, passing through its center of mass. Considering as $\rho$ the water density, the U-tube having a cross-section area $S$, much shorter than the tube length, $S_o$ the hole dimension, $L_h$ the length of the horizontal part of the tube, $L_v$ the height of the liquid surface in equilibrium in the tube columns, $F$ is the damping force associated with energy dissipation due to liquid passing through the orifice, $x$ is the horizontal displacement and $y$ is the vertical displacement. Let $T$ be kinetic energy and $V$ be potential energy. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L} = T - V = T_1 + T_2 + T_3 + T_4 - V_1 - V_2 - V_3 - V_4\,, \tag{1}$$

where the sub-index 1 is related to the vertical column on the left, 2 with the right vertical column, 3 with the horizontal column and 4 with the structure (box and U-tube). These components have the following expressions.

$$T_1 = \frac{1}{2}m_1\dot{y}^2 + \frac{1}{2}m_1\dot{x}^2 = \frac{1}{2}\left(\dot{y}^2 + \dot{x}^2\right)\rho S\left(L_v - y\right)\,; \tag{2}$$

$$T_2 = \frac{1}{2}m_2\dot{y}^2 + \frac{1}{2}m_2\dot{x}^2 = \frac{1}{2}\left(\dot{y}^2 + \dot{x}^2\right)\rho S\left(L_v + y\right)\,; \tag{3}$$

$$T_3 = \frac{1}{2}m_3\left(\dot{y} + \dot{x}\right)^2 = \frac{1}{2}\rho S L_h\left(\dot{y} + \dot{x}\right)^2\,; \tag{4}$$

$$T_4 = \frac{1}{2}M\,\dot{x}^2; \tag{5}$$

where $M$ is the mass of the box and the tube and $m$ is the mass of liquid inside the tube.

We also have to

$$V_1 = m_1 g h_1 = \rho S\left(L_v - y\right)g\frac{1}{2}\left(L_v - y\right) = \frac{1}{2}\rho S g\left(L_v - y\right)^2\,; \tag{6}$$

$$V_2 = m_2 g h_2 = \rho S\left(L_v + y\right)g\frac{1}{2}\left(L_v + y\right) = \frac{1}{2}\rho S g\left(L_v + y\right)^2\,; \tag{7}$$

$$V_3 = 0\,; \tag{8}$$

$$V_4 = \frac{1}{2}kx^2\,. \tag{9}$$

The Lagrangian of the system then has the form

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2}\left(\dot{y}^2 + \dot{x}^2\right)\rho S\left(L_v - y\right) + \frac{1}{2}\left(\dot{y}^2 + \dot{x}^2\right)\rho S\left(L_v + y\right) + \frac{1}{2}\rho S L_h\left(\dot{y} + \dot{x}\right)^2 + \frac{1}{2}M\,\dot{x}^2\\ &-\frac{1}{2}\rho S g\left(L_v - y\right)^2 - \frac{1}{2}\rho S g\left(L_v + y\right)^2 - \frac{1}{2}kx^2\,.\end{aligned} \tag{10}$$

We determine the equation of motion through the Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) - \frac{\partial \mathcal{L}}{\partial y} = F_y\,, \tag{11}$$

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where $F_y = \lambda\,\dot{y}$ is a non-conservative force in the $y$ direction; this force results from the dissipation of kinetic energy in the hole located in the center of the horizontal section of the tube. We can rewrite the Lagrange equation as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} + \frac{\partial V}{\partial y} = F_y\,.2\rho S L_v \ddot{y} + \rho S L_h \ddot{y} + \rho S L_h \ddot{x} + 2\rho S g y = -\lambda \dot{y}\,,$$

or

$$\rho S L \ddot{y} + \rho S L_h \ddot{x} + 2\rho S g y = -\lambda \dot{y} \tag{12}$$

where $L = 2L_v + L_h$.

In the $x$ direction, we have

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0\,,$$

or

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial V}{\partial x} = 0\,.$$

Substituting, we get

$$2\rho S L_v \ddot{x} + \rho S L_h \left(\ddot{y} + \ddot{x}\right) + kx + M\ddot{x} = 0$$

$$\left(\rho L S + M\right)\ddot{x} + \rho S L_h \ddot{y} + kx = 0. \tag{13}$$

We thus obtain the following system

$$\begin{cases} \rho S L \ddot{y} + \rho S L_h \ddot{x} + 2\rho S g y + \lambda \dot{y} = 0 \\ \left(\rho L S + M\right)\ddot{x} + \rho S L_h \ddot{y} + kx = 0 \end{cases} \tag{14}$$

The equation system presented in Eq. (14) can be solved in different ways, such as substitution method, Laplace transforms, and numerical methods. In the next section, the Laplace transform was chosen. This approach is justified given that the initial conditions are directly inserted into the system, which is unnecessary to find a solution and then apply them.

# 3 Solving the System of Ordinary Differential Equations

From the Eq. (14)

$$\begin{cases} (M+m)\ddot{x} + m'\ddot{y} + kx = 0, \\ m'\ddot{x} + m\ddot{y} + 2\rho S g y + \lambda \dot{y} = 0 \end{cases} \tag{15}$$

where $m = \rho\,L\,S$, $m' = \rho\,S\,L_h$, and the initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$, $y(0) = 0$, and $\dot{y}(0) = 0$ , we take the Laplace transform in both equations. Then, the system is rewritten as

$$\begin{cases} (M+m)\,\mathrm{L}\{\ddot{x}\} + m'\,\mathrm{L}\{\ddot{y}\} + k\,\mathrm{L}\{x\} = 0 \\ m'\,\mathrm{L}\{\ddot{x}\} + m\,\mathrm{L}\{\ddot{y}\} + 2\rho S g\,\mathrm{L}\{y\} + \lambda\,\mathrm{L}\{\dot{y}\} = 0 \end{cases} \tag{16}$$

knowing that

$$\begin{cases} \mathrm{L}\{x\} = X(s),\, \mathrm{L}\{\dot{x}\} = s\,\mathrm{L}\{x\} - x(0),\, \mathrm{L}\{\ddot{x}\} = s^2\,\mathrm{L}\{x\} - s\,x(0) - \dot{x}(0), \\ \mathrm{L}\{y\} = y(s), \mathrm{L}\{\dot{y}\} = s\,\mathrm{L}\{y\} - y(0),\, \mathrm{L}\{\ddot{y}\} = s^2\,\mathrm{L}\{y\} - s\,y(0) - \dot{y}(0). \end{cases} \tag{17}$$

 

where the variable $s$ is associated to the transform kernel $e^{-st}$. From (3), (16) and (17) we obtain

$$\begin{cases}[(M+m)s^2+k]X(s)+m's^2Y(s)=(M+m)sx_0\\ m's^2X(s)+[ms^2+2\rho Sg+\lambda s]Y(s)=m'sx_0\end{cases}. \tag{18}$$

Multiplying the first equation by $m's^2$, the second by $[(M+m)s^2+k]$ and subtracting

$$m'^2s^4Y(s)-[(M+m)s^2+k][ms^2+2\rho Ag-\lambda s]Y(s)=m'(M+m)s^3x_0-[(M+m)s^2+k]m'sx_0.$$

Simplifying the signs

$$[(m'^2-Mm-m^2)s^4-(M+m)\lambda s^3-[(M+m)2\rho Ag+km]s^2-k\lambda s-2k\rho Ag]Y(s)=-km'sx_0$$

$$Y(s)=\frac{-fs}{as^4+bs^3+cs^2+ds+e}, \tag{19}$$

where

$$\begin{cases}a=m'^2-Mm-m^2, b=-(M+m)\lambda, c=-[(M+m)2\rho Ag+km],\\ d=-k\lambda, e=-2k\rho Ag, f=-km'x_0.\end{cases} \tag{20}$$

Writing the 4$^{\text{th}}$ grade polynomial in factored form, we have

$$Y(s)=\frac{fs}{(s-r_1)(s-r_2)(s-r_3)(s-r_4)},$$

where $r_i$, $i=1,2,3$ e 4 are the polynomial roots. Now we can calculate the inverse Laplace transform:

$$y(t)=\int_0^\infty e^{st}Y(s)ds. \tag{21}$$

Substituting (19) found in (18), we have

$$X(s)=\frac{(M+m)x_0s}{[(M+m)s^2+k]}-\frac{m'fs^3}{A_1s^6+B_1s^5+c_1s^4+D_1s^3+E_1s^2+F_1s+G_1},$$

where

$$\begin{cases}A_1=(M+m)a, B_1=(M+m)b, C_1=(M+m)c+ka\\ D_1=(M+m)d+kb, E_1=(M+m)e+kc, F_1=kd, G_1=ke.\end{cases}$$

To find $x(t)$, we take the inverse Laplace transform of $X(s)$. We have then

$$x(t)=\int_0^\infty e^{st}X(s)ds. \tag{22}$$

In the next section, we discuss the results obtained by Eq. (22) and Eq. (35).

# 4 Results

Figures 2 and 3 illustrate the behavior of the primary structure and the fluid using the parameters: $M=8\,\mathrm{kg}$, $m=4\,\mathrm{kg}$, $k=150\,\mathrm{N/m}$, $A=0.005\,\mathrm{m}^2$, $m'=0.2\,\mathrm{kg}$, $g=9.8\,\mathrm{m/s}^2$, $\rho=1\,\mathrm{kg/m}^3$ e $\lambda=2\,\mathrm{kg/s}$.

As can be expected, the amplitude of the structure's (blue curve) movement in time and the fluid's (red curve) oscillation amplitude will be reduced. This is the expected behavior of the structure when subjected to oscillations, such as when an absorber is attached to it and inflicts a

 

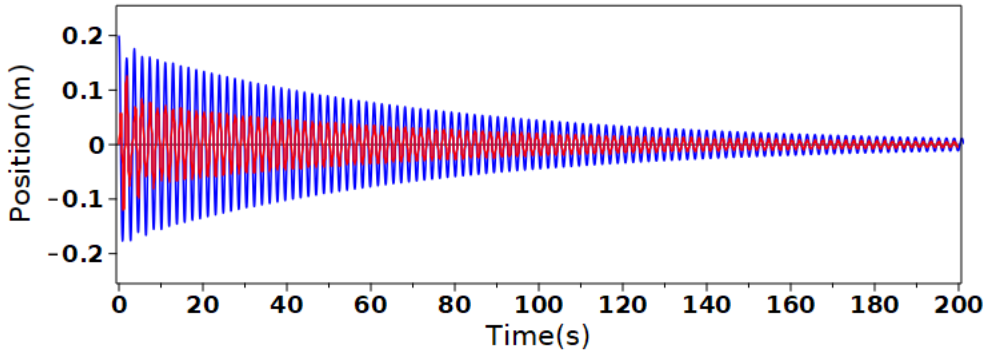


Figure 2: Damping of the structure (blue curve) and the fluid (red curve). Source: authors.

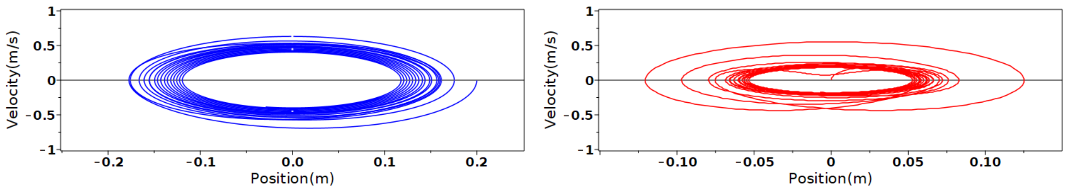


Figure 3: Phase space of the structure (blue curve) and the fluid (red curve). Source: by the authors.

loss of energy on it. This result confirms the idea of using TLCD to preserve structures that suffer undesirable movements due to, for instance, earthquakes and wind action. Figure 3, in particular, presents the system's phase space, which describes the temporal evolution of the structure and fluid through the temporal evolution of its dynamic variables, position, and velocity.

The model described in this work proved robust enough to allow for a more realistic description and to guide an experimental setup.

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