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Agricultural Subsidy/Tax in an Intersectoral Economic Model with Distinct Population Growth Rates

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Abstract. In this paper we introduce an *ad valorem* subsidy/tax to the agricultural sector in a two-sector – agricultural and industrial – migration and economic growth model. We show that the adoption of an agricultural subsidy (tax) by the government slows down (accelerates) the growing over time of the **per capita** capital of the economy, and of the proportion of workers in the industrial sector. Besides, we show that the adoption of a subsidy (tax) in the agricultural sector implies in a less (more) industrialized economy in the long run.

Keywords. Two-Sector Economic Growth Model, Labor Migration, Distinct Population Growth Rates, Agricultural Subsidy/Tax.

1 Introduction

The Mas-Colell and Razin two-sector migration and economic growth model [6] describes an economy composed by an industrial and an agricultural sectors, with perfect and instantaneous mobility of capital, but an imperfect and slow migration of labor between sectors. Such a model was used to explain the observed patterns of decreasing rate of migration from rural (agricultural) to urban (industrial) areas, as well as a stage of accelerated accumulation of capital, during the development of an economy. A modification of this model was used to study the implications of declining population growth rates in regional migration [2], and recently the authors proposed a generalization of the original model [6], allowing for distinct intersectoral population growth rates [3, 7], hypothesis that reflects better the available empirical data, with rural regions usually showing a distinct population organic growth rate than urban regions, where the industrial sector tends to be located [1, 2, 4, 5].

The main objective of this paper is to introduce, following [6], an *ad valorem* subsidy/tax to the agricultural sector in the model with distinct intersectoral population growth rates proposed in [3], and to analyse the impact of the adoption of such a governmental policy in the **per capita** capital of the economy, as well as in the proportion of workers employed in the industrial sector, especially in the long run.

This short paper is structured as follows: after this introduction, in section 2 we present the proposed model; in section 3 we obtain the steady state of the model, and the impact that the adoption of a subsidy/tax in the agricultural sector has in the equilibrium; in section 4 we present some numerical results; and in section 5 we close with our conclusions.

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2 The Model with Subsidy/Tax in the Agricultural Sector

The model proposed by [6] describes a two-sector economy composed by an industrial sector I, and an agricultural sector A, where there is a perfect and instantaneous mobility of capital, but an imperfect and slow migration of labor between sectors. Defining ρ as the fraction of the labor force employed in the industrial sector at time $t > 0$, assuming full employment of labor, and Cobb-Douglas production functions in both sectors, the **per capita** outputs in sectors I, y_I, and A, y_A, are given by:

$$y_I = \rho k_I^{\beta},\ y_A = (1-\rho)k_A^{\alpha}, \tag{1}$$

where $\alpha, \beta \in (0,1)$, and k_I, k_A are the **per capita** capital used in each sector. Supposing capital full employment, and defining $k > 0$ as the availability of **per capita** capital in the whole economy at $t > 0$, the following identity must be satisfied at all times:

$$\rho k_I + (1-\rho)k_A = k. \tag{2}$$

2.1 Instantaneous Intersectoral Equilibrium

At any time $t > 0$, the economy is characterized by a given distribution of labor force between sectors, ρ, and a given quantity of available **per capita** capital, k. While the capital market is in equilibrium between sectors at all times, due to the perfect and instantaneous mobility of capital, this is not necessarily the case in the labor market. Although the labor market inside each sector is always in equilibrium, it is not necessarily in equilibrium between sectors, since the migration of labor is not instantaneous. But it eventually reaches such equilibrium in the long run, as workers slowly migrate from one sector to another, equalizing the wage rates.

Starting with the analysis of the capital market, consider the agricultural good, that is completely consumed, as the **numéraire** of the economy, p as the price of the industrial good, which can be consumed or invested, and τ as the *ad valorem* subsidy/tax rate to the agricultural sector. If $\tau > 0$, the government gives a subsidy τ for each unity of produced agricultural good, in such a way that the price received by the producers is given by $p_A = 1 + \tau > 1$; if $-1 < \tau < 0$, then each unity of agricultural good is taxed by τ, and the price received by the producers becomes $p_A = 1 + \tau < 1$; finally, if $\tau = 0$, the model analysed in [3] is recovered, with $p_A = 1$.

The equalization of the marginal productivity of capital between sectors gives the equilibrium in the capital market:

$$p\beta k_I^{\beta-1} = (1+\tau)\alpha k_A^{\alpha-1}. \tag{3}$$

The equilibrium between supply and demand in the industrial good market is given by:

$$(s+\delta)y = py_I, \tag{4}$$

where:

$$y = py_I + y_A, \tag{5}$$

is the **per capita** income in the whole economy, $s \in (0,1)$ is the fraction of income spent in industrial goods for investment purposes, and $\delta \in (0,1)$ is the fraction of the income that is being spent in the industrial good for consumption purposes. Note that the total proportion of income that is spent in industrial goods is given by $(s+\delta) \in (0,1)$, while $[1-(s+\delta)] \in (0,1)$ is the proportion of the income spent in the agricultural good[3].

[3] We must have $s + \delta < 1$ in order to guarantee that the agricultural sector remains always active, i.e., that $1-(s+\delta) > 0$.

Considering the equations above, [6] show that the equilibrium in the capital market at any instant of time $t > 0$, is given by:

$$k_I = \theta_\tau \frac{k}{\rho},\ k_A = (1-\theta_\tau)\frac{k}{(1-\rho)}, \tag{6}$$

where θ_τ is defined as:

$$\theta_\tau = \frac{\beta(s+\delta)}{\beta(s+\delta)+\alpha(1-s-\delta)(1+\tau)} \in (0,1). \tag{7}$$

As for the labor market, assuming perfect competition in each sector, the equilibrium wage rates at the industrial, w_I, and agricultural, w_A, sectors are given, at any time, by:

$$w_I = p(1-\beta)k_I^\beta, w_A = (1+\tau)(1-\alpha)k_A^\alpha, \tag{8}$$

which may instantaneously differ.

2.2 Dynamics of the Model

For the dynamics of the economy's **per capita** capital, k, we propose that the organic population growth rates in the industrial and agricultural sectors are given by n_I and n_A, respectively, which may be different. This implies that:

$$\frac{\dot{k}}{k} = \lambda\theta_\tau^\beta \left(\frac{\rho}{k}\right)^{1-\beta} - \left[\rho n_I + (1-\rho)n_A\right],\ k(0) = k_0, \tag{9}$$

where:

$$\lambda = \frac{s}{s+\delta} \in (0,1), \tag{10}$$

is defined as the fraction of the total industrial output that is invested to create new capital goods, and $k_0 > 0$ is the initial level prescribed for the **per capita** capital. Note that, since $\rho \in (0,1)$, a weighted average of the population growth rates in both sectors is present in the RHS of the differential equation in (9), what slows down the increase of k. Also, if we make $n_I = n_A = n$ in (9), we recover the dynamics for k of the original model [6]. Besides, if $\tau = 0$, the model proposed in [3] is recovered.

The introduction of specific population growth rates for each sector also impacts the dynamics of the proportion of the total labor force in the industrial sector, ρ, which now is given by:

$$\frac{\dot{\rho}}{\rho} = m + (1-\rho)(n_I - n_A),$$

where $m = \frac{M}{L_I}$ is the relative migration rate into the industrial sector, M is the corresponding rate of migration (workers per period of time), and $L_I > 0$ is the current population in the industrial sector. As in the original model, we consider that workers migrate to the sector paying the highest wage rate, such that:

$$m = \gamma\,(w-1) = \sigma_\tau \frac{(1-\rho)}{\rho} - \gamma, \tag{11}$$

where w is the relative wage rate between sectors, defined as $w = \frac{w_I}{w_A}$, $\gamma > 0$ is a parameter giving the velocity of this migration, and σ_τ is defined as:

$$\sigma_\tau = \gamma\frac{(1-\beta)}{\beta}\frac{\alpha}{(1-\alpha)}\frac{\theta_\tau}{(1-\theta_\tau)} > 0. \tag{12}$$

Then, closing the model, the dynamics for ρ is given by the following initial value problem:

$$\frac{\dot{\rho}}{\rho} = \sigma_\tau \frac{(1-\rho)}{\rho} - \gamma + (1-\rho)(n_I - n_A),\ \rho(0) = \rho_0, \tag{13}$$

where $\rho_0 \in (0,1)$ is the initial proportion of the labor force in the industrial sector. Note that, when $n_I > n_A$ ($n_I < n_A$), the term $(1-\rho)(n_I - n_A)$ has a positive (negative) effect in the increase of ρ. If $n_I = n_A = n$, the problem (13) reduces to the dynamics of the original model presented in [6]. Furthermore, if $\tau = 0$ is considered in (13), the model proposed in [3] is recovered.

Note that from equations (7) and (12) we have that $\frac{\partial \theta_\tau}{\partial \tau} < 0$ and $\frac{\partial \sigma_\tau}{\partial \tau} < 0$. This implies that an increase in τ causes a slow down in the accumulation of capital governed by (9), $\frac{\dot{k}}{k}$, and a decrease in the relative migration rate into the industrial sector by (11), m, what also slows down the growing of the proportion of the total labor force employed in the industrial sector by equation (13), $\frac{\dot{\rho}}{\rho}$. In other words, the adoption of an agricultural subsidy by the government slows down the growing of $k(t)$ and $\rho(t)$ over time, while, on the other hand, the adoption of an agricultural tax has the opposite effect, acceleration the growing of the **per capita** capital of the economy, and of the number of workers in the industrial sector.

Remark: The system (9) and (13) gives $k(t)$ and $\rho(t)$ for all $t \geq 0$. Then, with this information in hand, it is possible to obtain the corresponding instantaneous equilibrium values for all quantities derived in section 2.1: $k_I(t)$, $k_A(t)$ [equation (6)], $y_I(t)$, $y_A(t)$ [equation (1)], p [equation (3)], $y(t)$ [equation (5)], and $w_I(t)$, $w_A(t)$ [equation (8)].

3 Steady State of the Model and Effects of the Subsidy/Tax

Setting the right hand side of equations (13) and (9) to zero, we obtain the steady state value of the proportion of the total labor force employed in the industrial sector, ρ_∞, and of the **per capita** capital implied by the model, respectively. The proofs presented in [3] can be easily adapted to show that the dynamic model given by equations (9) and (13) has a unique and stable economic feasible steady state, result that is summarized in Propositions 1 and 2 below.

Proposition 1 (steady state for ρ): The only feasible and stable steady state for the proportion of the total labor force employed in the industrial sector implied by the model (9) and (13), $0 < \rho_\infty < 1$, is the following:

$$\rho_\infty = \begin{cases} \dfrac{1}{2\Delta n}\left[-(\sigma_\tau + \gamma - \Delta n) + \sqrt{(\sigma_\tau + \gamma - \Delta n)^2 + 4\Delta n \sigma_\tau}\right], \text{ if } \Delta n \neq 0 \\ \dfrac{\sigma_\tau}{\sigma_\tau + \gamma}, \text{ if } \Delta n = 0 \end{cases} \tag{14}$$

where $\Delta n = n_I - n_A$.

Proposition 2 (steady state for k): The only economically feasible and stable steady state for the **per capita** capital implied by the model (9) and (13), $k_\infty > 0$, is given by:

$$k_\infty = \begin{cases} \rho_\infty \left(\dfrac{\lambda \theta_\tau^\beta}{n_A + \rho_\infty \Delta n}\right)^{\frac{1}{1-\beta}} = \rho_\infty \left(\dfrac{\lambda \theta_\tau^\beta}{\rho_\infty n_I + (1-\rho_\infty) n_A}\right)^{\frac{1}{1-\beta}}, \text{ if } \Delta n \neq 0 \\ \dfrac{\sigma_\tau}{\sigma_\tau + \gamma}\left(\dfrac{\lambda \theta_\tau^\beta}{n}\right)^{\frac{1}{1-\beta}}, \text{ if } \Delta n = 0 \end{cases} \tag{15}$$

where $\Delta n = n_I - n_A$, and ρ_∞ is given by (14).

Remark: In order to guarantee that k_∞ is a real number, we must have $n_A + \rho_\infty \Delta n > 0$. From now on we will consider that this condition is always satisfied.

Remark: If $\tau = 0$, the steady state presented in [3] are recovered. Besides, if $n_I = n_A = n$ ($\Delta n = 0$), Propositions 1 and 2 give the same steady state (k_∞, ρ_∞) of the original model presented in [6].

Below we present the main results of the present work, deriving the impact that marginal changes in the subsidy/tax rate to the agricultural sector, τ, have in the steady state values of the proportion of the total labor force employed in the industrial sector (ρ_∞), as well as in the **per capita** capital of the economy (k_∞), ceteris paribus.

Proposition 3 (effect of changes in τ on ρ_∞): If the subsidy/tax rate to the agricultural sector, τ, increases, the proportion of the total labor force employed in the industrial sector at the steady state, ρ_∞, decreases, and vice-versa. That is:

$$\frac{\partial \rho_\infty}{\partial \tau} < 0. \tag{16}$$

Proof: If $\Delta n = 0$, from (14) we get that $\frac{\partial \rho_\infty}{\partial \sigma_\tau} = \frac{\gamma}{(\sigma_\tau + \gamma)^2} > 0$, and since $\frac{\partial \sigma_\tau}{\partial \tau} < 0$, the result follows. If $\Delta n \neq 0$, also from (14) we have that:

$$\frac{\partial \rho_\infty}{\partial \sigma_\tau} = \frac{1}{2\Delta n} \frac{(\sigma_\tau + \gamma + \Delta n) - \sqrt{(\sigma_\tau + \gamma - \Delta n)^2 + 4\Delta n \sigma_\tau}}{\sqrt{(\sigma_\tau + \gamma - \Delta n)^2 + 4\Delta n \sigma_\tau}}. \tag{17}$$

If $\Delta n > 0$, then $(\sigma_\tau + \gamma + \Delta n) > 0$, and if we assume that the numerator in (17) is non-positive, we conclude that $\gamma \leq 0$, what contradicts the hypotesis that $\gamma > 0$. Therefore, the numerator must be positive, and then $\frac{\partial \rho_\infty}{\partial \sigma_\tau} > 0$. In case $\Delta n \leq -(\sigma_\tau + \gamma) < 0$, then the term $(\sigma_\tau + \gamma + \Delta n)$ in the numerator of (17) is also negative, and again we get that $\frac{\partial \rho_\infty}{\partial \sigma_\tau} > 0$. Finally, if we consider $-(\sigma_\tau + \gamma) < \Delta n < 0$, $(\sigma_\tau + \gamma + \Delta n) > 0$, and we can also show by contradiction that the numerator of (17) is negative, since $\gamma > 0$. Summarizing all the cases above, the result follows □

Proposition 4 (effect of changes in τ on k_∞): If $\Delta n < 0$, then if the subsidy/tax rate to the agricultural sector, τ, increases, the **per capita** capital of the economy at the steady state, k_∞, decreases, and vice-versa. That is:

$$\frac{\partial k_\infty}{\partial \tau} < 0. \tag{18}$$

Proof: If $\Delta n = 0$, from (15) we get that:

$$\frac{\partial k_\infty}{\partial \theta_\tau} = \frac{\partial \rho_\infty}{\partial \theta_\tau} \left(\frac{\lambda \theta_\tau^\beta}{n} \right)^{\frac{1}{1-\beta}} + \frac{\beta}{(1-\beta)} \frac{\lambda}{n} \theta_\tau^{\beta-1} \rho_\infty \left(\frac{\lambda \theta_\tau^\beta}{n} \right)^{\frac{\beta}{1-\beta}},$$

what is positive by Proposition 3, by the fact that $\frac{\partial \sigma_\tau}{\partial \theta_\tau} > 0$, that $\beta, \rho_\infty \in (0,1)$, and that all other parameters are positive. Since $\frac{\partial \theta_\tau}{\partial \tau} < 0$, we then get that $\frac{\partial k_\infty}{\partial \tau} < 0$ in this case. For $\Delta n \neq 0$, (15) implies that:

$$\begin{aligned} \frac{\partial k_\infty}{\partial \theta_\tau} &= \frac{\lambda k_\infty^\beta}{(1-\beta)(n_A + \Delta n)^2} \left(\frac{\theta_\tau}{\rho_\infty} \right)^\beta \left[\beta \left(\frac{\rho_\infty}{\theta_\tau} - \frac{\partial \rho_\infty}{\partial \theta_\tau} \right) (n_A + \rho_\infty \Delta n) + n_A \frac{\partial \rho_\infty}{\partial \theta_k} \right] \\ &> \frac{\lambda k_\infty^\beta}{(1-\beta)(n_A + \Delta n)^2} \left(\frac{\theta_\tau}{\rho_\infty} \right)^\beta \left[(1-\beta) n_A - \beta \rho_\infty \Delta n \right] \frac{\partial \rho_\infty}{\partial \theta_k} \\ &> 0, \text{ provided } \Delta n < 0, \end{aligned}$$

and we get the desired result □

Remark: From the proof of Proposition 4 above, we also have that $\frac{\partial k_\infty}{\partial \theta_\tau} > 0$, provided $0 < \frac{\theta_\tau}{\rho_\infty}\frac{\partial \rho_\infty}{\partial \theta_\tau} < 1$, and this implies that $\frac{\partial k_\infty}{\partial \tau} < 0$ for some interval of positive Δn. The numerical results in the next section suggests that this is the case for the particular set of parameters considered.

Economically, the results of Propositions 3 and 4 mean that if the government gives a subsidy to the agricultural sector, this implies in lower levels for the **per capita** capital of the economy, and for the number of workers in the industrial sector in the long run, i.e. in a less industrially developed economy when compared with the case with no agriculture subsidy. On the other hand, if the government taxes the agricultural sector, this implies in higher levels of both the **per capita** capital of the economy, and the number of workers in the industrial sector, in the long run, and consequently the economy ends up with a stronger industrial sector.

4 Some Numerical Results

In the Figure 1 we show the steady states ρ_∞ and k_∞ of the model, as functions of the subsidy/tax rate, τ, for $-1 < \tau < 1$, considering the following three scenarios for the intersectoral population growth rates: (i) $n_I = 0.01 < 0.05 = n_A$ ($\Delta n = -0.04 < 0$), (ii) $n_I = 0.05 = n_A$ ($\Delta n = 0$), and (iii) $n_I = 0.09 > 0.05 = n_A$ ($\Delta n = 0.04 > 0$). Following [6, 7], we have used the following constant theoretical values for the parameters of the model in all scenarios: $\alpha = 0.3$, $\beta = 0.4$, $s = 0.15$, $\delta = 0.6$, and $s = 0.15$. As we can see in the figure, in all these cases the unique economically feasible stable steady state for the proportion of the total labor force employed in the industrial sector (ρ_∞), as well as for the **per capita** capital of the economy (k_∞), are decreasing functions of the subsidy/tax rate, τ, what exemplifies the results of Propositions 3 and 4, even for the scenario (iii) where $\Delta n > 0$.

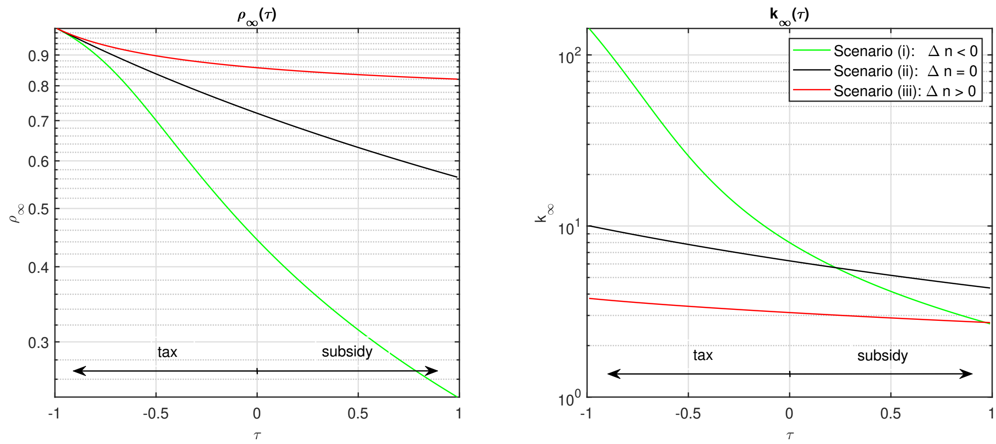

Figure 1: Steady states $\rho_\infty(\tau)$ and $k_\infty(\tau)$ for scenarios: (i) $n_I = 0.01 < 0.05 = n_A$ ($\Delta n = -0.04 < 0$), (ii) $n_I = 0.05 = n_A$ ($\Delta n = 0$), and (iii) $n_I = 0.09 > 0.05 = n_A$ ($\Delta n = 0.04 > 0$)

5 Conclusions

In this work we have introduced an *ad valorem* subsidy/tax to the agricultural sector in the model with distinct intersectoral population growth rates proposed in [3], in order to analyse the impact of the adoption of such a governmental policy in the steady state of the model.

We have shown that the adoption of an agricultural subsidy by the government slows down the growing over time of the **per capita** capital of the economy, and of the proportion of workers in the industrial sector, while, on the other hand, the adoption of an agricultural tax has the opposite effect. Besides, we have proved that the **per capita** capital of the economy, and the proportion of the total labor force employed in the industrial sector at the steady state, are both decreasing functions of the subsidy/tax rate to the agricultural sector, τ. This means that if the government gives a subsidy to the agricultural sector, this implies in lower levels for the **per capita** capital of the economy, and for the proportion of workers employed in the industrial sector in the long run, i.e. in a less industrially developed economy when compared with the case with no subsidy. On the other hand, if the government taxes the agricultural sector, this implies in higher levels of both the **per capita** capital of the economy and the proportion of workers in the industrial sector in the long run, and consequently the economy ends up with a stronger industrial sector.

Future research may consider the introduction of different production functions, logistic population growth, imperfect capital mobility between the sectors, and technological progress in the model.

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