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A brief mathematical model for yellow fever

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The threat of an infectious disease's outbreak, which may cause severe consequences around the globe, is a cause for concern to the whole world. In Brazil, for instance, the country is trying to deal with the risk of the Yellow Fever. According to the World Health Organization (WHO) in the most recent outbreak, dating back to 2017, about 200 deaths caused by the disease were registered in Brazil's southeast region alone. This situation brings into question if it's possible to get relevant mathematical models that could help to predict the infection's dissemination in a given population.

Mathematical Epidemiology and its compartmentals models are useful tools to undestand epidemic outbreaks. An Ordinary Differentional Equations System is used to represent the varying rate of all the system's compartments in a time series. The study of the system's linear and structural stability has a major importance in understanding the infection's behavior. Therefore, the goal of this work is to build a simple model of a Yellow Fever epidemic process and analyse its stability and bifurcation points.

The created model to be studied refers to the populational interactions of the vectors (mosquitoes), as it follows:

$$\begin{cases} \dfrac{dE}{dt} = \phi_m.f.M(t) - q.\sigma_e.E(t) & \dfrac{dP}{dt} = \sigma_l.L(t) - \sigma_p.P(t) \\ \dfrac{dL}{dt} = q.\sigma_e.E(t) - \sigma_l.L(t) & \dfrac{dM}{dt} = \sigma_p.P(t) - \mu_m.M(t) \end{cases} \tag{1}$$

Where E represents the eggs, L the larvae, P the pupae and M, the mosquitoes populations. As can be seen, we have a linear, autonomous, non-conservative, coupled system. The used parameters can be seen in details on the following table:

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Table 1: Summary of mosquito population parameters and its units.

Symbol	Meaning	Unit
f	fraction of eggs originatin female mosquitoes	-
q	fraction of eggs hatching to larva stage	-
σ_e	per-capita transition rate, from egg to larva	day^{-1}
σ_l	per-capita transition rate, from larva to pupa	day^{-1}
σ_p	per-capita transition rate, from pupa to mosquito	day^{-1}
μ_m	per-capita mortality rate of mosquito	day^{-1}
ϕ_m	oviposition rate per female mosquito	day^{-1}

This phenomenological model only has a trivial equilibrium point, without the presence of mosquitoes: $(E, L, P, M) = (0, 0, 0, 0)$. The stability analysis reveals its linear instability. To determine the equilibrium point, we have to assume that $\phi_m f \neq \mu_m$ otherwise, we will have infinite solutions to the system used to calculate the equilibrium points as well as a null eigenvalue. This assumption led to simulations using the values of the parameters obtained from [3], and as a result, when $\phi_m f < \mu_m$, each eigenvalue obtained is negative (or its real part is negative) resulting in a stable equilibrium point; and when $\phi_m f > \mu_m$ three negatives eigenvalues were obtained (or their real part is negative) and one is positive breaking the stability. These results were expected, since, $\phi_m f$ is strongly related to the origin of new mosquitoes and μ_m is related to the disappearance of the population, hence in the first case, the population converges back to the equilibrium point, but in the second case, it grows indefinitely.

From now on, several simulations will be executed, varying the main parameters which influences directly the epidemic process in order to analyze the different behaviors of the disease's spread. A new version for the model will be studied, considering competition among some compartments (logistic terms), adding non-linear terms as well as time-varying parameters, therefore, a non-linear and non-autonomous model.

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