Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, Vol. 2, N. 1, 2014.
Trabalho apresentado no CMAC-Sul, Curitiba-PR, 2014.

Linear Matrix Inequalities Control Driven Applied to Bimorph Piezoelectric Energy Harvesting

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Abstract: *Harnessing energy from environment requires maximum interaction between harvesters and energy power source. Since available energy in natural system is random and small power, harvester must increase their capability. Is known that resonant solution for energy harvesting devices can improve their performance, however projects are limited for a small time matching natural frequency offered from environment vibration. To enhance energy harvesting result it is propose a controller to driven harvester to maximum vibration interaction to the power source. This study is based in numerical evaluation. The proposed controller regards Linear Matrix Inequalities (LMI) H∞ approach. As main result it is noted a considerable increase of system vibration energy available for harvesting.*

Introduction

Several studies concerning Energy Harvesting arises at academic frontier. Among this many enforces to advance in harnessing energy from environment emerge studies in solar, thermic gradients and vibration as power source to electric energy conversion.

Centering in vibration, also there are several methods to transform kinetic to electric energy, for example piezoelectric materials, electromagnetic and electrostatic transducers [6]. Piezoelectric direct effect transform mechanic tension applied to material into electricity. Instead firstly application in sensor and actuators piezoelectric direct effect is growing in studies for power generation applications, since better piezoelectric materials has been developed [11].

Using piezoelectric materials concerning direct effect to supply electric power was not possible since few years ago because of small power generation resulting in this application. Nevertheless, electronic devices are in direction to low power consumption and it becomes a reality to ponder in piezoelectric direct effect as power source [10].

Recently energy harvesters using piezoelectric material were proposed in numerous project designs. First projects concentrate enforces to design seeking to match the device natural frequency to harmonic vibration frequency resulting in resonance and, consequently enhance power transducing [9]. However, environment vibration is in a large range, therefore only a part of space time becomes into resonance, reducing harvesters projected efficiency [8]. Because this drawback there are various studies concentrate in increase harnessing efficiency, for example innovating piezoelectric material [2], use of tuning devices [2], setting non-linear bistable behavior [7] and modeling non-ideal excitation [3].

Finally, this paper explores a controller to set energy harvester to maximum vibration frequency interaction. It is possible using an active controller with vibrational actuation according exogenous excitation, due an adequate mathematic treatment of dynamic model. This research proposes to use Optimum Linear Matrix Inequalities method [1] to control a bimorph energy harvester seeking to enhance efficiency.

Bimorph Energy Harvester

A bimorph cantilever excited by a harmonic vibration [4] is given in Figure 1, including an electrical load circuit.

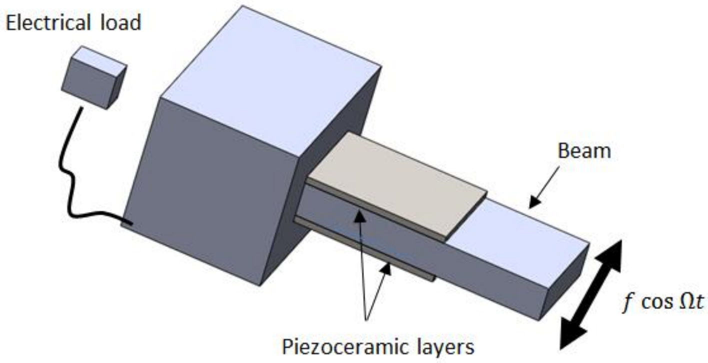

Figure 1 - Bimorph cantilever [4].

For the proposed harvester the dimensionless open loop parameters are given in Table 1, regarding resonance response.

Open loop Parameters	ζ	χ	Λ	κ
Value	0.01	0.05	0.05	0.5

Table 1 - Dimensionless parameters [4].

Dynamic Model

The dynamic model for the energy harvester is given in dimensionless Equation 1, where ζ is damping, χ is piezoelectric mechanical coupling, ν is resistance voltage, Λ is reciprocal of time constant, k is piezoelectric electric coupling and w is the exogenous excitation [4].

$$\begin{aligned} &\ddot{x} + 2\zeta\dot{x} + x - \chi v = w \\ &\dot{v} + \Lambda v + k\dot{x} = 0 \end{aligned} \qquad (1)$$

The space-state matrix form of the dynamic system is given by Equation 2:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2\zeta & \chi \\ 0 & -\kappa & -\Lambda \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} w \qquad (2)$$

LMI Control Driven

An LMI Control Driven utilizes feedback state, u(t) = Lx(t), seeking optimize relation between exogenous excitation w(t) and result signal y(t) [1]. The space-state model for the proposed system is given by Equation 3, and explained in schematic flow chart in Figure 2.

$$\begin{cases} \dot{\mathrm{x}} = Ax + B_1 w + B_2 u \\ \mathrm{y} = \mathrm{Cx} \end{cases} \qquad (3)$$

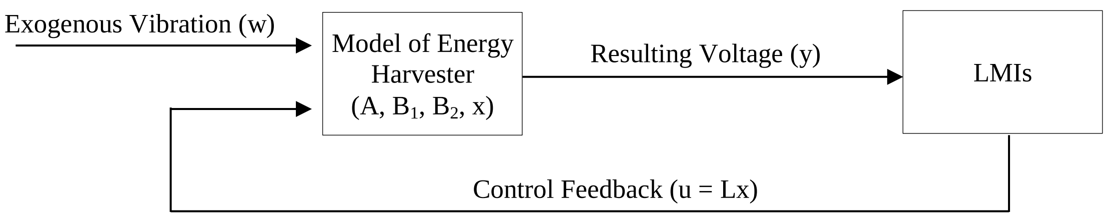

Figure 2: Dynamic control scheme

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An sufficient condition for LMI control driven [1] is the existence of a matrix $X = X' \in \mathcal{R}^{nxn}$ and $Y \in \mathcal{R}^{mxn}$, presented by LMIs (Linear Matrix Inequalities) given in Equation 4:

$$\begin{cases} \min \mu \\ \begin{bmatrix} AX + XA' - B_2Y - Y'B_2' & XC' + Y'D' & B_1 \\ CX + DY & -I & 0 \\ B_1' & 0 & -\mu I \end{bmatrix} < 0 \\ X > 0 \end{cases} \tag{4}$$

with j = 1, ..., r

When LMIs are feasible, a feedback matrix L for space-state that optimizes system behavior is given for $L = YX^{-1}$.

Controller Project

Substituting parameters given in Table 1 than the related matrix according Equation 2 are given in Equation 5.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -0.02 & 0.05 \\ 0 & -0.5 & -0.05 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \ B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\ and\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{5}$$

Substituting matrix A, B1, B2 and C given in Equation 5 in LMIs given in Equation 4 and solving inequalities, the resulting feedback matrix is $L = 10^5[\,1.2630 \quad 0.0007 \quad 0.0000]$. The resulting eigenvalues are -0.0808 + 1.0373i, -0.0808 - 1.0373i and -0.0438 + 0.0000i. Since all eigenvalues have real part negative, consequently the system is stable. Once L is determined, the feedback system configuration is $u = -Lx$ and $y = A - B_2Lx$.

Frequency Response Analysis

As explained in paper introduction, the environment vibration is in wide range and it is not possible to project design according resonance. Applying a controller it is possible to set up to maximum frequency response as presented in Figure 3.

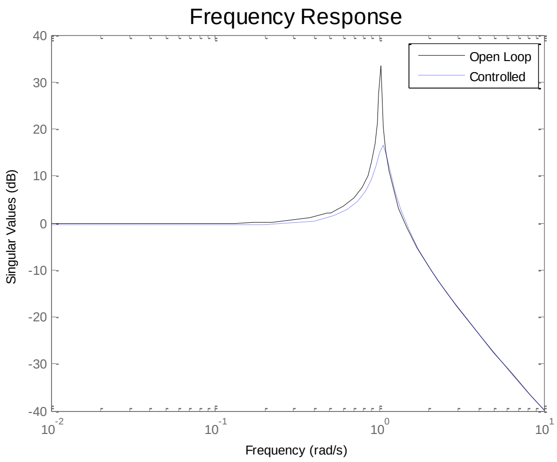

Figure 3: System frequency response

In Figure 3 is shown that the controller gives a response closely to open loop system. Notice that the open loop corresponds to dimensionless resonance parameters.

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Efficiency Analysis

Considering the dimensionless dynamic model proposed by [4], a resonance occurs for f = 0.083 and Ω = 0.8. The dynamic system given in Equation 1 is written $\ddot{x} + 2\zeta\dot{x} + x - \chi v = 0.083\cos 0.8t$. Velocity, displacement and acceleration have direct effect to output voltage as can be seen in phase portrait analysis [3]. Taking in consideration the feedback state matrix A-B_2L, it is possible to calculate the feedback parameters, as shown in Table 2.

Feedback parameters	ζ	χ	Λ	κ
Value	0.0778	0.2636	0.1636	0.6356

Table 2 - Feedback parameters.

For evaluation of controller efficiency was set up different values for force varying from 0.083 as resonance to 0.3. Performing analysis according Runge-Kutta method for solving ordinary differential equations it is possible to visualize the total energy from the system, given in Figure 5, 6 and 7.

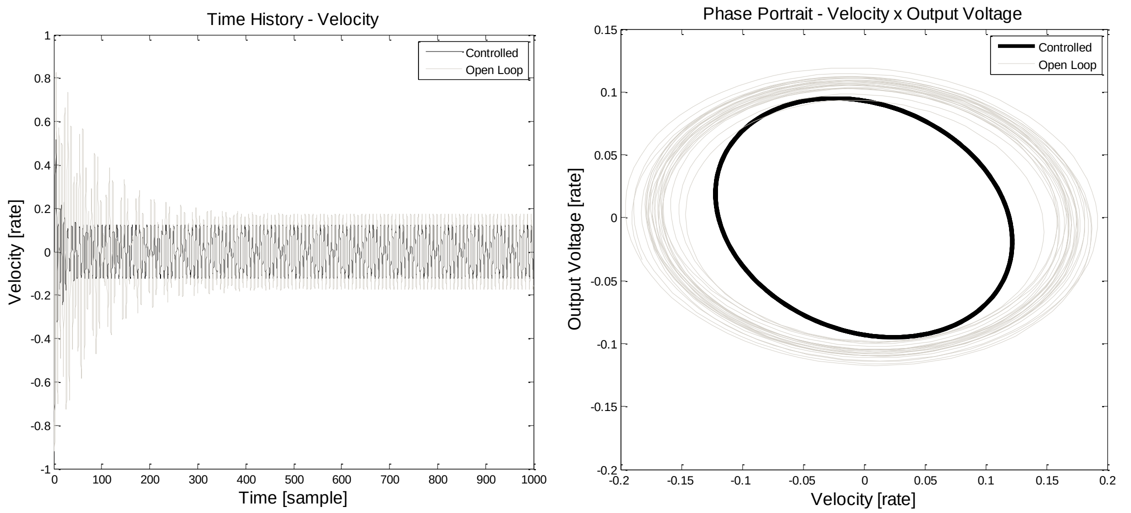

Figure 5: Available System Energy - f = 0.083

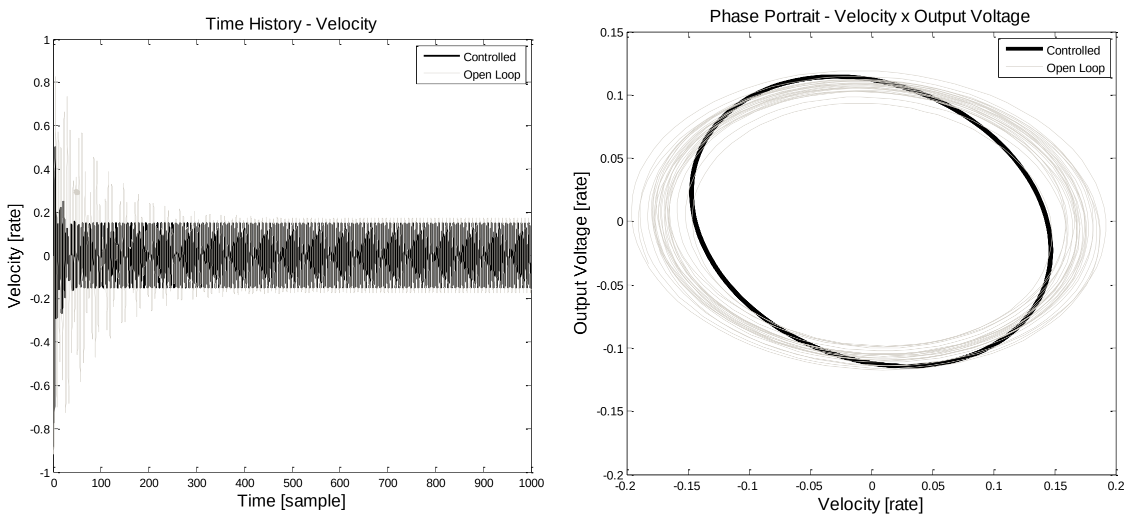

Figure 6: Available System Energy - f = 0.1

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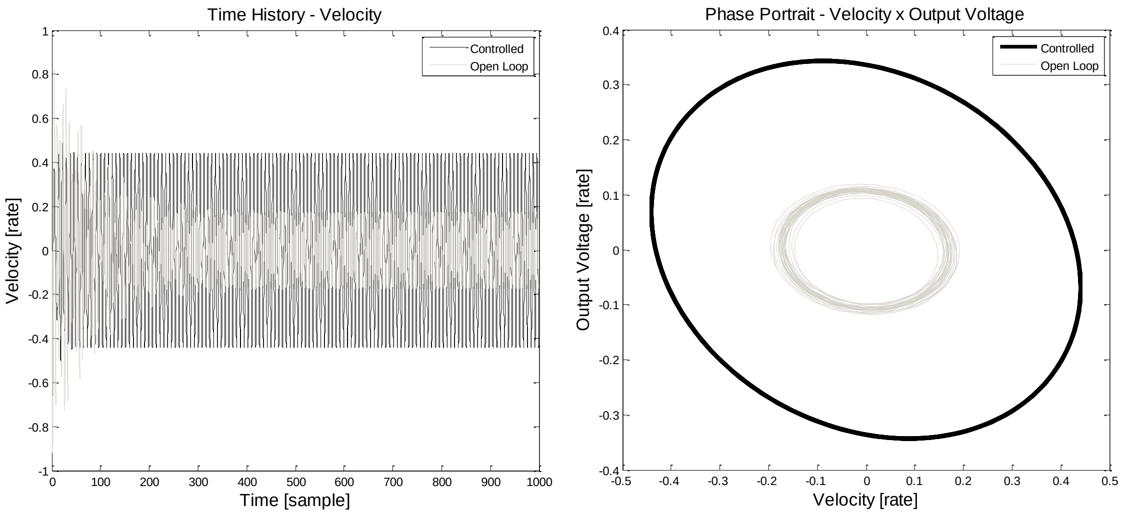

Figure 7: Available System Energy - f = 0.3

As can be seen in Figure 5, the amount of energy available for controlled system is smaller than resonance open loop but very closely. For f = 0.1 the energy from both system are similar, as shown in Figure 6. However, when force increases and the system get apart to resonance, the total energy from the controlled system becomes higher then open loop system. For f = 0.3, for example, the total energy from controlled system becomes significantly superior to open loop resonance system, as shown in Figure 7.

Final Remarks

Various studies regarding energy harvesting has been outcome in the last few years to enhance the harness capability from energy harvesters. In the same direction, this investigation presented a study of control driven for dynamic behavior considering Linear Matrix Inequalities Optimum H∞ method. As the main result, it was possible to verify a significant increase of system energy available for harvesting.

In this study the parameters were considered singular, however in real case situation, damping and electrical coupling depends of environment conditions, for example humidity and temperature. Variation in parameters results in polytopic uncertainties. In this direction, instead optimum method is required robust LMI method [5] which characterizes a possible research continuation.

Acknowledgements

This is a preliminary study from PhD in Mechanical Engineering at UNESP University. Control driven for non-ideal and chaotic behavior, considering polytopic parameters is the study continuation proposing new solutions for energy harvesting project design.

The first two authors would like to thank to CNPq and Minas Gerais State Research Foundation (FAPEMIG) for funding through the National Institute of Science and Technology in Smart Structures (INCT-EIE).

Reference

[1] Andrea, C. Q., Pinto, J. O. P., Assunção, E., Teixeira, M. C. M., Galotto, L. J., “Controle ótimo H∞ de sistemas não-lineares com modelos fuzzy Takagi-Sugeno”, Controle & Automação, Vol.19 no.3, Julho, Agosto e Setembro, 2008.

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[2] Challa V. R., Prasad, M. G., Shi Y., Fisher F. T., "A vibration energy harvesting device with bidirectional resonance frequency tenability". Smart Materials and Structures, No. 17, 2008.

[3] Chavarette F. R., "Control design applied to a non-ideal structural system with behavior chaotic", International Journal of Pure and Applied Mathematics, Vol. 86 No. 3, 487-500, 2013.

[4] Erturk, A. and Inman, D., "Broadband piezoelectric power generation on high-energy orbits of the bistable duffing oscillator with electromechanical coupling", Journal of Sound and Vibration, Vol. 330, No. 10, pp. 2339-2353, 2011.

[5] Faria, F. A. "Projetos de controladores baseados em LMI usando realimentação da derivada dos estados", Tese de Doutorado, PPGEE UNESP, 2009.

[6] Harb, A., "Energy harvesting: State-of-the-art", Renewable Energy, 36, 2641-2654, 2011.

[7] Harne R. L., Wang K. W. (2013), "A review of the recent research on vibration energy harvesting via bistable systems", Smart Materials and Structures, No. 22

[8] Miller, L. M., Halvorsen, E., Dong, T., Wright, P. K., "Modeling and experimental verification of low-frequency MEMS energy harvesting from ambient vibrations", Journal of Micromechanics and Microengineering, No. 21, 13p, 2011.

[9] Moehlis, J., DeMartini, B. E., Rogers, J. L., Turner, K. L., "Exploiting Nonlinearity to Provide Broadband Energy Harvesting", Proceedings ASME Dynamic Systems and Control Conference, October 12-14, Hollywood, California, USA, 2009.

[10] Sodano H. A., Inman D.l J., "Comparison of Piezoelectric Energy Harvesting Devices for Recharging Batteries", Journal of Intelligent Material Systems and Structures, 16(10), 799-807, 2005.

[11] Yeager, C. B., Trolier-McKinstry, S., "Epitaxial Pb(Zrx,Ti12x)O3 ($0.30 \leq x \leq 0.63$) films on (100)MgO substrates for energy harvesting applications", Journal of Applied Physics 112, 074107, 2012.

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