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Application of an Adaptive GMRES(m) on an Electromagnetic Scattering Problem from a Cavity

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1 Introduction

In this work an adaptive GMRES(m) solver is applied to an electromagnetic scattering problem from a cavity embedded in a ground plane [3,4]. The problem is focused on a 2-D geometry by assuming that the medium is invariant in the z-direction and nonmagnetic with constant magnetic permeability $\mu(x, y) = \mu_0$. The ground plane (x axis) and the wall of the cavity are perfect electric conductors, and the interior of the cavity is filled with inhomogeneous material characterized with its relative permittivity $\epsilon_r(x, y)$.

For a TM (transverse magnetic) polarization, in which the magnetic field is transverse to the invariant direction and the electric field is $E = (0, 0, u(x, y))$, the approximation to the cavity problem consists of the discrete Helmholtz equation (1) together with a Sommerfeld's radiation condition imposed at infinity -see equation (3)-, in order to reduce the problem to the following bounded-domain problem:

$$\Delta u + k_0^2 \epsilon_r u = f, \text{in } \Omega = [0, a] \times [b, 0], \tag{1}$$

$$u = 0, \text{on } S, \tag{2}$$

$$\partial_n u = \mathcal{T}(u) + g, \text{on } \Gamma \tag{3}$$

where k_0 is the free space wave number, Ω is the problem domain, f is the source term and $f = 0$ in the free space, S denotes the walls of cavity, $\mathcal{T}$ is a nonlocal boundary operator, Γ is the aperture between the cavity and the free space and $g(x) = -2i\beta e^{i\alpha x}$.

2 Methodology

GMRES(m) is frequently selected for its robustness in problems whose discretization results in a large sparse non-hermitian linear system [5]. GMRES(m) approximates the

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solution of the subsequent linear system $Au = f$ at each cycle j as follows [2]:

$$z_j = GMRES(A, r_{j-1}, m) \quad (4)$$

$$u_j = u_{j-1} + z_j \quad (5)$$

$$r_j = f - Au_j \quad (6)$$

where u_j is the approximate solution at the cycle. When its main parameter m is fixed, the convergence is not guaranteed, causing possibly stagnation [5].

An adaptive control strategy based on switching between at least two Krylov subspace methods is performed. The search subspace is enriched with information vectors of previous cycles by using error approximation vectors in an augmented GMRES(m). If stagnation occurs, the latest error approximation vectors are discarded because they do not help to avoid the stagnation, hence we return to standard GMRES(m) increasing the restart parameter m by a constant. The new approximate solution is found in a $(m + k)$-dimensional subspace, constructed by the Krylov Subspace using the new restart parameter and augmented with k recently generated error approximations. Generally, $k \leq 3$ and performance decreases for larger k [1]. For this work, we choose $k = 3$.

3 Conclusion

Numerical experiments for different discrete domain sizes and values of wave number k_0 are compared. Preliminary results show that a switching strategy is good enough to improve convergence when comparing with other iterative methods with fixed parameters. However, the computation is specially challenging when k_0 is increasing.

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