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Mathematical Model for the ZIKA Epidemic using Ordinary Equations and with Temporary Delay

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Abstract. Zika virus is primarily transmitted to people through the bite of an infected mosquito from the Aedes genus, mainly Aedes aegypti in tropical regions. Aedes mosquitoes usually bite during the day, peaking during early morning and late afternoon/evening. This is the same mosquito that transmits dengue, chikungunya and yellow fever.
Zika is transmitted to humans primarily through bites from infected Aedes aegypti and Aedes albopictus mosquitoes. The transmission is in both directions, that is, infected mosquitoes infect humans and infected humans infect mosquitoes. Sexual transmission of Zika virus is also possible. Other modes of transmission such as blood transfusion are being investigated. In this work are presented mathematical models to predict the behavior of the ZIKA epidemic over time, using ordinary differential equations [6, 14] and ordinary differential equations with temporal delay [11]. The delay is the time it takes humans and mosquitoes to develop the virus, become infected and participate in the transmission dynamics [9, 11]. The computer simulations are performed for Suriname and El Salvador, which have different characteristics, Suriname have higher rate the infection of human to mosquito and El Salvador have from mosquito to human, and this allows us to adapt the models to different epidemiological conditions. The data of the parameters and initial conditions were extracted from [5, 12, 13, 16].

Keywords. Delay, model, simulations, Zika.

1 Introduction

Zika fever (also known as Zika virus disease) is an illness caused by the Zika virus. The disease is spread through the bite of daytime-active Aedes mosquitoes such as the A. aegypti and A. albopictus (these are the same mosquitoes that spread dengue and chikungunya viruses). Its name comes from Zika forest in Uganda, where the virus was first isolated from a rhesus monkey in 1947. The first human cases were reported in Nigeria in 1954. The first documented outbreak among people occurred in 2007, in the Federated State of Micronesia [15].

The disease of Zika virus is transmitted from infected Aedes mosquitoes to humans

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through mosquito bites [9]. It can also be transmitted from human to human through the blood and semen of an infected human, and through an infected pregnant woman to the foetus. Zika is a cause of microcephaly and other severe brain defects [5]. The incubation period (the time from exposure to symptoms) of Zika virus disease is not clear, but is likely to be a few days to a week. The symptoms are similar to other arbovirus infection such as dengue, and include fever, skin rashes, conjunctivitis (red eyes), muscle and joint pain, malaise and headache. These symptoms are usually mild and usually last from 2- 7 days [9].

There is no specific treatment or vaccine currently available for Zika virus disease. Prevention and control relies on reducing mosquitoes through source reduction (removal and modification of breeding sites), and reducing contacts between mosquitoes and people.

The use of ODE (ordinary differential equation) and ODE with delay in the study of epidemics can be seen in [2,13], in particular for Dengue in [12,17], for HIV/AIDS in [1,18], for Ebola in [4,7] and Zika in [5,15,16], these texts contributed as background in the work that we present.

The objective of this work is to present models for the Zika epidemic based on ODE and ODE with delay. A theoretical study of the model was made and the $\Re_0$ was calculated for the submodel with only contagion by mosquitoes and with only sexual contagion. Computational simulations are performed in Surinam and El Salvador, where Zika can become endemic. We performed a comparison between the two variants of modeling with respect to the time of the epidemic and the number of infected.

2 Transmission Model

The model variables are susceptible men H_s, susceptible women M_s, exposed men H_E, exposed women M_E, infected men H_I, infected women M_I, recovered men H_R, recovered women M_R, susceptible mosquitoes V_s and infected mosquitoes V_I. The parameters of the model are between 0 and 1 and are described in Table (1).

For the construction of the model, there is immunity in the recovered state, the infected

Table 1: Description of parameters used in the model.

Parameters	Description
β_{y_1}	The force of infection from infected mosquito to susceptible human
β_{y_2}	The force of infection from infected man to susceptible man
β_{y_3}	The force of infection from infected man to susceptible woman
β_x	The force of infection from infected human to susceptible mosquito
μ_1, μ_2, η	Man, woman and mosquito natural death rates
ϵ_1, ϵ_2	Disease- induced death rate for humans (man and woman)
r_1, r_2	Per capital recovery rate for humans from the infectious (man and woman)
N_1	Entry rate of susceptible men
N_2	Entry rate of susceptible women
N_3	Entry rate of susceptible mosquitoes

man can infect a woman and a susceptible man, result of study of other epidemics that are transmitted by sexual contact, the death by natural causes is equal in any state, the

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death of mosquitoes will be due to environmental factors because no control strategy is applied and by definition epidemiological $H_s, M_s, H_E, M_E, H_I, M_I, H_R, M_R, V_s, V_E$ and V_I are continue and positive or null.

The transmission dynamics of the ZIKA is modeled by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dH_s}{dt} &= N_1 - \beta_{y_1} V_I H_s - \beta_{y_2} H_I H_s - \mu_1 H_s,\\
\frac{dM_s}{dt} &= N_2 - \beta_{y_1} V_I M_s - \beta_{y_3} H_I M_s - \mu_2 M_s,\\
\frac{dH_E}{dt} &= \beta_{y_1} V_I H_s + \beta_{y_2} H_I H_s - (\omega_1 + \mu_1) H_E,\\
\frac{dM_E}{dt} &= \beta_{y_1} V_I M_s + \beta_{y_3} H_I M_s - (\omega_2 + \mu_2) M_E,\\
\frac{dH_I}{dt} &= \omega_1 H_E - (\epsilon_1 + \mu_1 + r_1) H_I,\\
\frac{dM_I}{dt} &= \omega_2 M_E - (\epsilon_2 + \mu_2 + r_2) M_I,\\
\frac{dH_R}{dt} &= r_1 H_I - \mu_1 H_R,\\
\frac{dM_R}{dt} &= r_2 M_I - \mu_2 M_R,\\
\frac{dV_s}{dt} &= N_3 - \beta_x H_I V_s - \beta_x M_I V_s - \xi V_s,\\
\frac{dV_E}{dt} &= \beta_x H_I V_s + \beta_x M_I V_s - (\omega_3 + \xi) V_E,\\
\frac{dV_I}{dt} &= \omega_3 V_E - \xi V_I. \qquad (1)
\end{aligned}$$

Initial Conditions

$$\begin{array}{lll}
H_s(0) = h_s > 0 & M_s(0) = m_s > 0 & H_I(0) = h_i > 0\\
M_I(0) = m_i > 0 & H_R(0) = h_r \geq 0 & M_R(0) = m_r \geq 0\\
H_E(0) = h_e \geq 0 & M_E(0) = m_e \geq 0 & V_s(0) = v_s > 0\\
V_I(0) = v_i > 0. & &
\end{array}$$

The disease- free equilibrium of the model is given as:

$$v_0 = \left(\frac{N_1}{\mu_1}, 0, 0, 0, \frac{N_2}{\mu_2}, 0, 0, 0, \frac{N_3}{\xi}, 0, 0\right).$$

The mosquito transmission route only model is obtained by assuming that virus is not trasmitted sexually. The reproduction number is given by:

$$\Re_0^m = \rho(-T\Sigma^{-1}) = \sqrt{k_1(v_0) + k_2(v_0)}, \qquad (2)$$

where $k_1(v_0) = \dfrac{\beta_{y_1} N_1 \beta_x N_3 \omega_1 \omega_3}{\mu_1 \xi^2 (\omega_1 + \mu_1)(\epsilon_1 + \mu_1 + r_1)(\omega_3 + \xi)}$ and $k_2(v_0) = \dfrac{\beta_{y_1} N_2 \beta_x N_3 \omega_2 \omega_3}{\mu_2 \xi^2 (\omega_2 + \mu_2)(\epsilon_2 + \mu_2 + r_2)(\omega_3 + \xi)}$.

Lemma 2.1. *The disease-free equilibrium is locally asymptocally stable if* $\Re_0^m < 1$, *and unstable if* $\Re_0^m > 1$ *for the sub-model with only mosquito transmission.*

The sexual transmission route only model is obtained by assuming that Zika virus is only transmitted sexually and not through the bites of infectious mosquitoes. The reproduction number is:

$$\Re_0^s = \rho(-T\Sigma^{-1}) = \frac{\beta_{y_2} H_s \omega_1}{(\omega_1 + \mu_1)(\epsilon_1 + \mu_1 + r_1)}. \qquad (3)$$

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Lemma 2.2. *The disease-free equilibrium is locally asymptocally stable if* $\Re_0^s < 1$*, and unstable if* $\Re_0^s > 1$ *for the sub-model with only sexual transmission.*

The $\Re_0^s$ and $\Re_0^m$ was calculated using the next-generation matrix method presented in [10]. The Lemma 2.1 and Lemma 2.2 are results of using Theorem 2 of [10] for the model.

Existence, uniqueness and positivity of the model solution were demonstrated [3,6,14].

2.1 Transmission Model with Delay

The mosquito becomes infected when it consumes the blood of a sick person. Then, if the insect bites a healthy person, it transmits the virus, which enters the bloodstream and is incubated for 3 or 12 days, until the symptom begins aparace. That time we will call it a delay and we will have it in the model by τ_1. The delay τ_2 will refer to the time that the mosquito that delays in developing the pathogen, 4 or 7 days [8,9]. The delay is taken into account in the infected compartment.

The transmission dynamics of the ZIKA taking into account the time delay is modeled by the system of differential equations with delay following:

$$
\begin{aligned}
\frac{dH_s}{dt} &= N_1 - \beta_{y_1} V_I H_s - \beta_{y_2} H_I H_s - \mu_1 H_s,\\
\frac{dM_s}{dt} &= N_2 - \beta_{y_1} V_I M_s - \beta_{y_3} H_I M_s - \mu_2 M_s,\\
\frac{dH_I}{dt} &= \beta_{y_1} V_I(t-\tau_2) H_s + \beta_{y_2} H_I(t-\tau_1) H_s - (\mu_1 + r_1 + \epsilon_1) H_I,\\
\frac{dM_I}{dt} &= \beta_{y_1} V_I(t-\tau_2) M_s + \beta_{y_3} H_I(t-\tau_1) M_s - (\mu_2 + r_2 + \epsilon_2) M_I,\\
\frac{dH_R}{dt} &= r_1 H_I - \mu_1 H_R,\\
\frac{dM_R}{dt} &= r_2 M_I - \mu_2 M_R,\\
\frac{dV_s}{dt} &= N_3 - \beta_x H_I(t-\tau_1) V_s - \beta_x M_I(t-\tau_1) V_s - \xi V_s,\\
\frac{dV_I}{dt} &= \beta_x H_I(t-\tau_1) V_s + \beta_x M_I(t-\tau_1) V_s - \xi V_I. \qquad (4)
\end{aligned}
$$

Initial Conditions

$$H_s(0), M_s(0), H_I(0), M_I(0), H_R(0), M_R(0), V_s(0), V_I(0) : [-\tau, 0] \longrightarrow \mathbb{R}_0^+.$$

For the models we demonstrate the existence, uniqueness and positivity of the solution, we study the stability and we found the R_0 [10, 11].

2.2 Discussion

The simulations are carried out for Suriname and El Salvador, which have different characteristics (in Suriname $\beta_{y_1} > \beta_x$ and in El Salvador $\beta_x > \beta_{y_1}$) and Zika can become an endemic problem. The data the parameters and initial conditions were extracted from [5,12,13,16] and Matlab-R2017a software was used for programming and the methods were extracted from [3, 11]. The unit of time in months because of the characteristics of the epidemic.

The study of $\Re_0$ for these scenarios showed that $\Re_0^m > 1$ (the infection will be able to

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spread in a population) and $\Re_0^s < 1$ (the infection will disappear in the long term), so the infection by mosquito bites has a strong influence on the dynamics.

According to model (1), El Salvador has more infections than Suriname, the rate of mosquito infection has a greater impact on the epidemic. El Salvador has the highest number of recovered and the number of infected men is greater than that of infected women in both countries.

With a time delay, the number of people infected decreases and the outbreak has a shorter duration compared to the model without delay. Here too, the number of infected men exceeds the number of infected women. Men recover more than women.
noindent In both scenarios it is true that more infected implies more recovered, see Figure 1 and Figure 2.

The models show for both scenarios, that the Zika will become an endemic problem and the result obtained in the calculation of the $\Re_0$ was evidenced, since only with the infection by mosquito bites will the infection spread to a population for an indeterminate time.

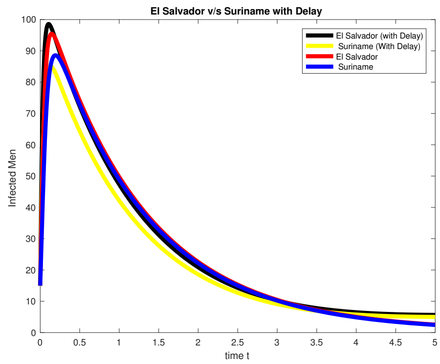

Figure 1: Infected men in Suriname and El Salvador with delay.

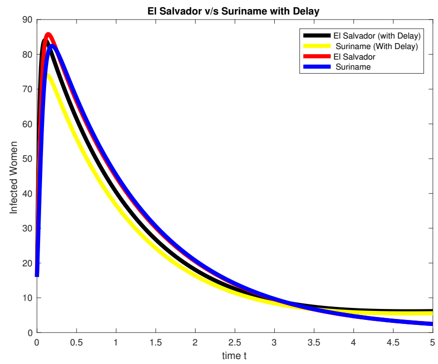

Figure 2: Infected women in Suriname and El Salvador with delay.

3 Conclusion

A mathematical model for the Zika epidemic is presented with an exposed variable and with temporal delay and a comparison was made between them in relation to the period of the outbreak and the number of infected. Computational simulations were performed for El Salvador and Suriname, their characteristics allow the study to be extended to other regions. The study showed that over time the Zika can become endemic and the study of the $\Re_0$ demonstrated the need for a control strategy with priority in the contacts between the mosquito and the human being. The infection rate of infected mosquito to human susceptible has a great influence on the dynamics. There is a strong relationship between the infection rate and the recovery rate, more infection means more recovered. With the delay, an outbreak is reported for a shorter period and less infected number.

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