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h-Adaptivity Applied to Ice Sheet Simulation

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Abstract. Marine ice sheets are large masses of glacial ice that terminate in the ocean forming an attached floating ice shelf. The region where the ice start to float is called grounding line. The evolution of the grounding line plays a major role in marine ice sheet dynamics, as they are a fundamental control of the marine ice sheet stability. Numerical modeling of grounding line dynamics requires significant computational resources and the accuracy of its position depends on grid or mesh resolutions. A technique that can improve accuracy with reduced computational cost is the adaptive mesh refinement approach. In recent years, this approach has been applied in ice sheet simulations, but the mesh refinement performed is not conducted by error estimators. Here, we implement and test the performance of the ZZ error estimator as a refinement criterion using the Ice Sheet System Model. In the numerical experiments carried out, the ZZ error estimator presents high values around grounding line and proves to be a good indicator of which elements should be refined. Our comparison results show that computational time using the ZZ estimator depends on the required accuracy, but for all cases, it is significantly smaller than the uniformly refined meshes cost.

Keywords. h-Adaptivity, Error estimator, Ice sheet simulation

1 Introduction

Marine ice sheets are large masses of glacial ice that terminate in the ocean forming a floating ice shelf [10]. The region where the ice start to float is the grounding line. A schematic geometry of a marine ice sheet is shown in Figure 1. The grounding line

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evolution plays a major role in marine ice sheet dynamics, as they are a fundamental control of marine ice sheet stability.

Numerical modelling of grounding line dynamics requires significant computational resources and the accuracy of their positions depends on grid or mesh resolutions. A technique that can improve accuracy with reduced computational cost is the adaptive mesh refinement (AMR) approach. Although some works in literature have used this technique, the refinement performed is not conducted by an associated error estimator.

Here, we apply a posteriori error estimator proposed by Zienkiewicz and Zhu [11] in the Ice Sheet System Model (ISSM) [6]. The AMR capability in ISSM relies on two different and independent meshers: Bamg and NeoPZ. Bamg is a bidimensional anisotropic mesh generator developed by Hecht [5] and embedded in ISSM. NeoPZ is a finite element library developed by Devloo [2] dedicated to high adaptive techniques. In this work, we use the NeoPZ package to perform the adaptive mesh refinement.

We test different refinement criteria based on the Zienkiewicz and Zhu (ZZ) error estimator: 1) error estimator calculated for the ice thickness, 2) error estimator calculated for the deviatoric stress tensor and 3) combination of criteria 1) and 2). We run the MISMIP+ [1] experiment using the Shelfy-Stream Approximation [7, 8] to compare the results obtained with both uniform and adpative meshes as well as the performance of each criterion in terms of grounding line position accuracy and computational time.

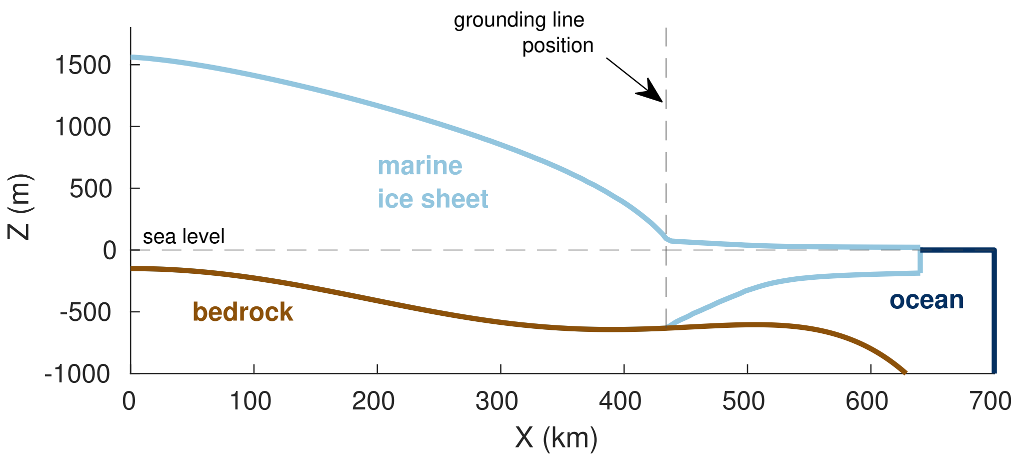

Figure 1: Schematic vertical plane view of a marine ice sheet and position of the grounding line.

2 Methods

This section describes the governing equations of the Shelfy-Stream Approximation and ice evolution, the ZZ error estimator implemented and the numerical experiment setup used to assess the error estimator.

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2.1 Governing equations

The ice sheet evolution is modelled by two sets of equations which are related to the mass and stress balances. In the Shelfy-Stream Approximation, the mass balance is represented by the ice thickness evolution:

$$\frac{\partial H}{\partial t} = -div\,(H\mathbf{v}) + \dot{m}_s \tag{1}$$

where H is the ice thickness, $\mathbf{v} = [v_x\ v_y]^{\mathbf{T}}$ is the vertically integrated velocity field and $\dot{m}_s$ is a source term (accumulation/ablation rate).

The stress balance of the Shelfy-Stream Approximation is vertically integrated and considers just the the xy-plane velocity field, v_x and v_y, as follows:

$$\begin{aligned} div\,(2H\bar{\mu}\dot{\boldsymbol{\varepsilon}}_x) - \alpha^2 v_x &= \rho g H \tfrac{\partial s}{\partial x} \\ div\,(2H\bar{\mu}\dot{\boldsymbol{\varepsilon}}_y) - \alpha^2 v_y &= \rho g H \tfrac{\partial s}{\partial y} \end{aligned} \tag{2}$$

where

$$H\bar{\mu} = \int_b^s \frac{1}{2}\frac{B}{\dot{\varepsilon}_e^{\frac{n-1}{n}}}dz \tag{3}$$

$$\dot{\boldsymbol{\varepsilon}}_x = \begin{bmatrix} 2\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \end{bmatrix} \quad \dot{\boldsymbol{\varepsilon}}_y = \begin{bmatrix} \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \\ \frac{\partial v_x}{\partial x} + 2\frac{\partial v_y}{\partial y} \end{bmatrix} \tag{4}$$

where $\bar{\mu}$ is the vertically integrated ice viscosity, α is the basal friction coefficient, ρ is the ice density ($918\,kg/m^3$), g is the gravitational acceleration, s is the ice surface, b is the ice base, B is the ice rigidity (temperature dependence), n is the Glen's law exponent [3] (usually taken as equal to 3) and $\dot{\varepsilon}_e$ is the effetive strain rate. The ice viscosity depends on the temperature and the effetive strain rate, but typically assumes values between 10^{13} to $10^{17}\,Pa\,s$ [4]. The terms $\alpha^2 v_x$ and $\alpha^2 v_y$ in Eqs. 2 are the basal friction contributions, which are null for the floating ice. The boundary conditions are defined as:

$$\begin{aligned} &\mathbf{v} = \mathbf{v}^D \text{ in } \mathit{\Gamma}_D \\ &2\bar{\mu}\dot{\boldsymbol{\varepsilon}}_x \cdot \mathbf{n} = \left(\tfrac{1}{2}\rho g H^2 - \tfrac{1}{2}\rho_w g b^2\right) n_x \text{ in } \mathit{\Gamma}_w \\ &2\bar{\mu}\dot{\boldsymbol{\varepsilon}}_y \cdot \mathbf{n} = \left(\tfrac{1}{2}\rho g H^2 - \tfrac{1}{2}\rho_w g b^2\right) n_y \text{ in } \mathit{\Gamma}_w \end{aligned} \tag{5}$$

where $\mathbf{n} = [n_x\ n_y]^{\mathbf{T}}$ is the outward-pointing unit normal vector, ρ_w is the water density ($1028\,kg/m^3$), $\mathit{\Gamma}_D$ is the Dirichlet contour and $\mathit{\Gamma}_w$ is ice-ocean contact contour (Neumann).

The dynamics of grounding line is implemented in ISSM through implicit level set functions, ϕ_{GL}, which is based on hydrostatic floatation criterion [9], such that:

$$\begin{aligned} &\phi_{GL} < 0\text{:} && \text{ice is floating,} \\ &\phi_{GL} > 0\text{:} && \text{ice is grounded,} \\ &\phi_{GL} = 0\text{:} && \text{grounding line position.} \end{aligned} \tag{6}$$

The resolutions of Eqs. 1 and 2 are performed through the standard approach of finite element method implemented in the ISSM [6].

2.2 ZZ error estimator

The generic form of the ZZ [11] error estimator e_k for a given element k is:

$$e_k = \left(\int_{\Omega_k} (\nabla u^* - \nabla u)^2 \, d\Omega_k \right)^{1/2} \tag{7}$$

where Ω_k is the domain of the element k, ∇u is the gradient of the Galerkin solution u and ∇u^* is the smoothed gradient. The smoothed gradient is calculated in each vertex i of the element k using:

$$\nabla u^* = \sum_{i=1}^{3} \phi_i \nabla u_i^* \tag{8}$$

and

$$\nabla u_i^* = \frac{1}{W} \sum_{j=1}^{n} w_j \nabla u_j \tag{9}$$

where ϕ_i is the shape function i, j is the jth element connected to the vertex i, w_j is the weight relative to the element j and W is the sum of all weights for the vertex i. We implement the ZZ error estimator for ice thickness (H) and deviatoric stress tensor (τ).

2.3 Numerical experiment

We run the third Marine Ice Sheet Model Intercomparison Project MISMIP+ [1] setup to compare the performance of each criterion in terms of grounding line position accuracy and computational cost. We run all numerical experiments starting from an initial configuration until a steady state condition is reached, which occurs in $20,000$ years.

3 Results

The spatial distributions of the ZZ error estimators for the deviatoric stress tensor, τ, and for the ice thickness, H, are shown in Figure 2. The position of the grounding line at steady state is also shown in Figure 2. It is notable that the error estimators calculated for τ present high values around the grounding line. For H, the distribution of high values is not confined in the region near the grounding line. The positions of the grounding line obtained using uniform and adaptive meshes are shown in Table 1. We run with 3 refinement criteria using the ZZ error estimator: 1) ZZ for H, 2) ZZ for τ and 3) ZZ for H and τ. We also run a combination of the estimators (H and τ) with a heuristic approach where elements near the grounding line are also refined. In this case, we use a distance to grounding line equal to $5\,km$ within which the elements are refined.

Table 1: Grounding line (**GL**) position at y=40 km for different meshes and refinement levels.

Refinement level and criterion	N^o **elements**	**GL position (km)**
Coarse	6,780	435.6
Level 1, uniform	27,120	447.1
Level 2, uniform	108,480	451.9
Level 3, uniform	433,920	456.3
Level 1, ZZ H	18,597	445.3
Level 1, ZZ τ	15,864	446.8
Level 1, ZZ H and τ	20,791	446.8
Level 2, ZZ H	41,350	449.5
Level 2, ZZ τ	20,891	452.6
Level 2, ZZ H and τ	46,869	451.2
Level 2, ZZ H and τ, and 5 km	47,694	451.8
Level 3, ZZ H	64,507	450.3
Level 3, ZZ τ	21,936	455.3
Level 3, ZZ H and τ	68,596	453.4
Level 3, ZZ H and τ, and 5 km	77,503	455.5

4 Discussion and conclusions

The ZZ error estimators calculated for ice thickness and deviatoric stress tensor present high values around grounding line. In particular, for ice thickness, the estimator also presents high values in the grounded part of the ice sheet: these follow the high gradient region of the bedrock topography.

Using a combination of the error estimators for both ice thickness and deviatoric stress tensor generates more elements than using individual estimator, as expected. Using the estimator for the deviatoric stress tensor produces accurate results in comparison to uniform meshes, but with less elements. The use of the estimator for ice thickness improves the bedrock geometry description, but it is not enough to generate same results in comparison to the other criteria. The combination of the estimators with the heuristic approach produces more elements in comparison to the others, but it is the most accurate among the criteria used here.

Our comparison analysis shows that computational time with adaptive mesh refinement depends on the required accuracy, but for all cases, it is significantly smaller than the uniformly refined meshes cost. To illustrate, for a 500-year run and time step of 1 year, the uniform mesh (level 3) takes approximately 4,800 seconds. For the same level and refining every time step, the adaptive mesh using ZZ for both ice thickness and deviatoric stress tensor takes 2,900 seconds. Both tests are performed in parallel (16 cores) in an Intel Xeon E5-2630 v3 2.40 GHz.

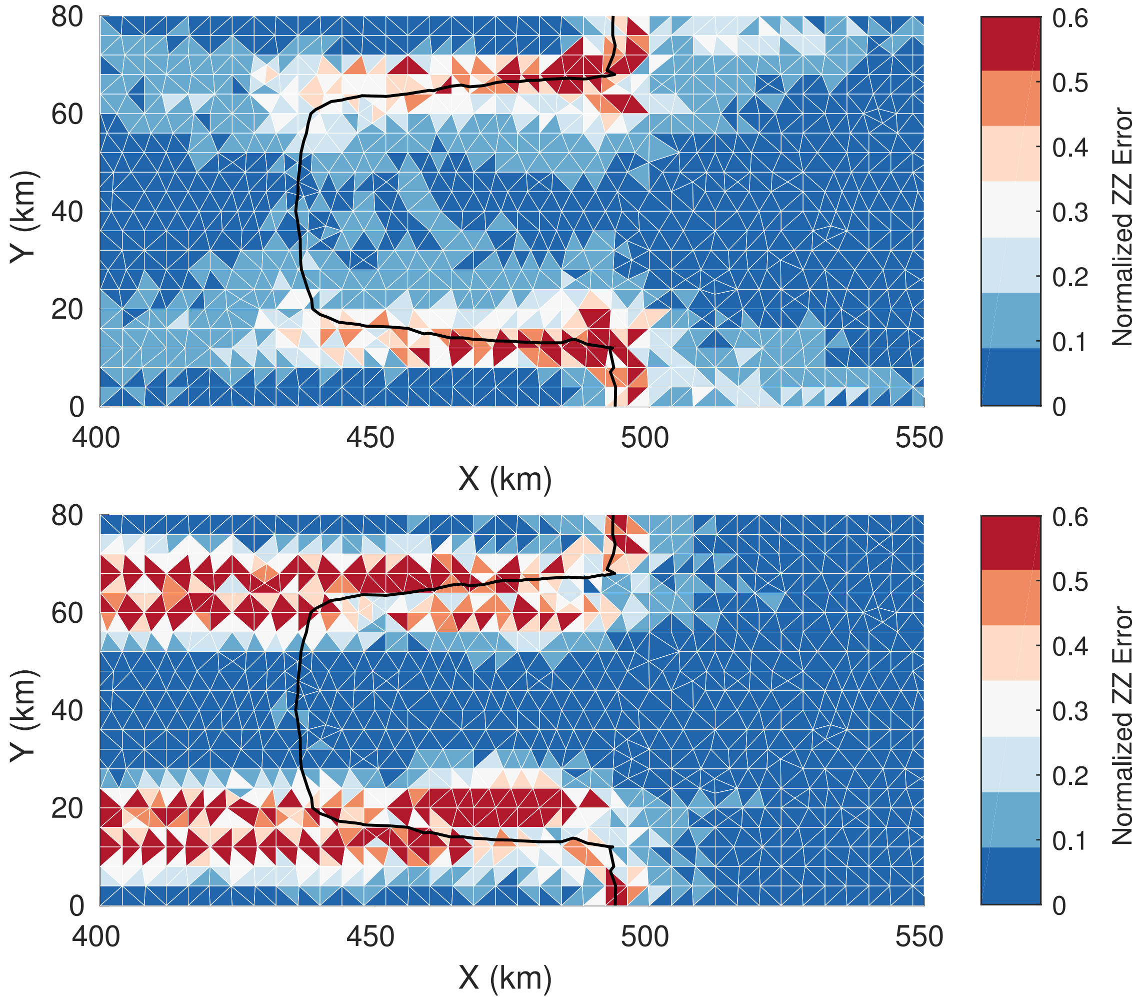

Figure 2: **Top**: spatial distribution of the ZZ error estimator calculated for the deviatoric stress, τ. **Bottom**: spatial distribution of the ZZ error estimator calculated for the ice thickness, H. **White lines**: MISMIP+ coarse mesh used as initial mesh for all experiments. **Black line**: grounding line position at steady state.

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