Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 6, n. 2, 2018.

Trabalho apresentado no XXXVIII CNMAC, Campinas - SP, 2018.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Analysis of the resonance effect on the orbital motion of a spacecraft around Mercury

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abstract. The double-averaged method should be used with caution in some situations where the averaging is applied at different timescales. In this work, a study is presented considering this observation for orbits around Mercury. When the average anomaly of the Sun is eliminated, the idea is that all effects whose periods below 88 days are neglected. As the rotation of Mercury is about 58.6 days, this means that the perturbation due to the C_{22} term must also be neglected. However, since the C_{22} term is important and should be taken into account, then terms longer than 58.6 days should also be preserved. In other words, keeping the C_{22} term with a period of 58.6 days means that the solar terms with the longest period (88 days) must also be maintained. Therefore, in this preliminary work, the second average over the mean anomaly of the Sun (or Mercury) is not applied. An analysis of the orbital motion of artificial satellites around Mercury is presented taking into account its non-sphericity (J_2, C_{22}) and the perturbation of the third body. The single-averaged method is applied to eliminate only the mean anomaly of the spacecraft. In this work, is presented an approach considering the Kozai resonance.

Key words. Astrodynamics, Orbital Perturbation, Kozai Resonance

1 Introduction

In Carvalho et al. [3] presents an analysis of the orbital motion of artificial satellites around Mercury taking into account its non-sphericity, the perturbation of the third body and the solar radiation pressure. Where the double-averaged method was applied to eliminate the short-period terms of the artificial satellite and the disturbing body, respectively. The motivation for the execution of this present work was based on the reference [3] and in the observations of one of the referees, which are highlighted in the following paragraph.

When the average anomaly of the Sun is eliminated all effects whose periods below 88 days are neglected. The rotation of mercury is about 58.6 days, so in principle effects with a period of less than 88 days should be eliminated. However, as the C_{22} term is important, see references [1,3,12], and therefore must be taken into account in the disturbing potential. This means that terms longer than 58.6 days should also be preserved. Thus, here in this work, the second average over the mean anomaly of the Sun (or Mercury) is not applied. The approach presented in this paper is based in the Kozai resonance. In reference [14] is

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DOI: 10.5540/03.2018.006.02.0250

presented a study about the orbits of the outer satellites of Jupiter. The authors showed an analytical expression for the libration of the pericenters. An analysis on resonance is made, especially for the libration of the apses and Kozai resonance. In reference [13] is analyzed effect of the secular resonance for the future of Phobos and Triton orbits. The author develops the equation of the disturbing potential of the third body using the single-averaged method. The non-sphericity (J_2) of the central body is also considered. The resonance when the period of precession of the longitude of the satellite pericenter is near 1 year is known as evection resonance (see reference [11]). According to reference [15] the evection resonance is caused by 1:1 commensurability between ϖ (longitude of the satellite pericenter) and $\lambda_{\odot}$ (Sun's mean longitude). An approach is presented in reference [11] where the authors investigated the probability of capture in evection resonance as a function of the rate of tidal evolution and initial eccentricity. Analytical expressions are developed for analysis of the resonant system, the escape mechanism of the evection resonance is explored. The evection resonance is also the subject of the interesting paper in reference [9] on the moons of extrasolar planets. In reference [9] it is shown that when a moon-hosting planet undergoes internal migration, the dynamic interactions can of course destroy the moon by capturing in a so-called evection resonance. In this resonance, the eccentricity of the lunar orbit grows until the moon eventually collides with the planet (see reference [4]). In reference [5] an analytical expansion of the disturbing function arising from direct planetary perturbations on the motion of satellites is derived. The authors have constructed an analytical model describing the evection resonance between the longitude of pericenter of the satellite orbit and the longitude of a planet, and study its dynamic. According to the authors, an interesting resonance happens when the secular frequency $\dot{\varpi}$ of the satellite is commensurable with n_p (mean motion of the planet), which corresponds to the argument $2w+2\Omega-2\lambda$. This approach is different from the classical one where classical evection resonance occurs between the longitude of the satellite pericenter and the longitude of the Sun (see reference [11]). In the continuation of this present work will be made a study considering this type of resonance (evection).

2 Disturbing Potential

In reference [15] is presented a simplified model that allows to derive the resonant configurations. The authors show, in an analytical way, the values of the semimajor axis where evection resonances can occur. The disturbing potential for the Jupiter-Sun system is developed in reference [15] considering the single-averaged model, where the disturbing body is in circular and planar orbit. Here, we consider a resonance model similar to that one shown in reference [15], but now we include the terms due to non-sphericity of the planet (J_2 and C_{22}). The bodies that are considered in this work are Mercury, Sun and spacecraft. Considering the Cartesian system fixed in Mercury. The reference plane is the equator of the planet. See figure 1 of the reference [15] for more details. The disturbing function of the motion of the artificial satellite perturbed by the Sun is

$$R_{\odot} = \frac{\mu_{\odot} r^2}{2r_{\odot}^3}(3cos^2(S) - 1) \tag{1}$$

DOI: 10.5540/03.2018.006.02.0250

where $\mu_\odot$ is the gravitational parameter of the Sun, r and $r_\odot$ are the position vector of the satellite and of the Sun, respectively. Here S is the angular distance between Sun and the satellite. Now using equation (2.3) of the reference [15] ($i_\odot = 0$) for the expression of S, we get

$$\begin{aligned}\cos(S) = [\cos(f+g)\cos(h) - \sin(f+g)\sin(h)\,c]\cos\left(\lambda_\odot\right) + \\ [\cos(f+g)\sin(h) + \sin(f+g)\cos(h)\,c]\sin\left(\lambda_\odot\right)\end{aligned} \tag{2}$$

where $\lambda_\odot = f_\odot + \varpi_\odot$, $\varpi_\odot$ is the longitude of pericenter of the Sun. The orbital parameters of the spacecraft are (a, e, i, l, g, h, f) semimajor axis, eccentricity, inclination, mean anomaly, argument of the pericenter, longitude of the node, and true anomaly, respectively. The same variables with index $\odot$ are used for the Sun. Here $c = cos(i)$. In this preliminary study the same disturbing potential of the reference [15] is considered, but developed differently. Here we do not use equations (2.5) and (2.6) of the reference [15], we made a change of variable to perform the average over the mean anomaly of the spacecraft, where a change in the integration variable is adopted for eccentric anomaly (E). This is done by using known equations from the celestial mechanics, which are:

$$\sin(f) = (\sqrt{1-e^2}\sin(E))/(1 - e\cos(E)); \tag{3}$$

$$\cos(f) = (\cos(E) - e)/(1 - e\cos(E)) \tag{4}$$

$$r/a = 1 - e\cos(E) \tag{5}$$

$$dl = (1 - e\cos(E))dE \tag{6}$$

Now replacing equations (2), (3), (4) and (5) in equation (1), the average is made using equation (6) and after algebraic manipulations we obtain,

$$\begin{aligned}R2_{SA} = -\tfrac{15}{16}\tfrac{\mu_\odot a^2}{a_\odot{}^3}(-1/2e^2(\cos(i)-1)^2\cos(-2h+2g+2\lambda_\odot) - \\ 1/2e^2(\cos(i)+1)^2\cos(2h+2g-2\lambda_\odot) + \\ 1/5(3e^2+2)(\cos(i)-1)(\cos(i)+1)\cos(2h-2\lambda_\odot) + \\ (\cos^2(i)e^2 - e^2)\cos(2g) - 1/5(3e^2+2)(\cos^2(i)-1/3))\end{aligned} \tag{7}$$

Now, with $i = 0$ and algebraically manipulating, we get

$$R2_{SA_{i=0}} = \tfrac{\mu_\odot a^2}{2a_\odot{}^3}[(\tfrac{15}{4})e^2\cos(2\varpi - 2\lambda_\odot) + \tfrac{1}{2}(\tfrac{3}{2}e^2+1)] \tag{8}$$

where $\varpi = h + g$ (longitude of the satellite pericenter). We find exactly the equation (3.4) of the reference [15]. Here $\varpi - \lambda_\odot$ is the critical angle studied in reference [15].

3 Non-sphericity of the central body

The development of this section is presented in reference [2] and will be replicated here. Considering the equatorial plane of the planet as the reference plane, the disturbing potential can be written in the form:

$$U = -\frac{\mu}{r}\left[\left(\frac{R_p}{r}\right)^2 J_2 P_2(\sin\phi) - \left(\frac{R_p}{r}\right)^2 C_{22} P_{22}(\sin\phi)\cos(2\gamma)\right], \tag{9}$$

where μ is the gravitational constant of the planet, R_p is the equatorial radius of the planet, P_n are the Legendre polynomials, P_{nm} the associated Legendre polynomials, the angle ϕ is the latitude of the orbit with respect to the equator of the planet, the angle γ is the longitude measured from the direction of the longest axis of the planet. Here the γ term contains the time explicitly (see reference [6]). Using spherical trigonometry we have $\sin\phi = \sin i \sin(f+g)$. The Legendre polynomials for J_2 and the Legendre associated functions for the sectorial C_{22} term can be written in the form (see reference [6])

$$\begin{aligned} &P_2(\sin\phi) = \tfrac{1}{2}(3s^2\sin^2(f+g)-1), \\ &P_{22}(\sin\phi)\cos 2\lambda = 6(\xi^2\cos^2 f + \chi^2\sin^2 f + \xi\chi\sin 2f) - 3(1-s^2\sin^2(f+g)), \end{aligned} \tag{10}$$

where we used the shortcut $\xi = \cos g\cos\Omega - c\sin g\sin\Omega$, $\chi = -\sin g\cos\Omega - c\cos g\sin\Omega$, $s = \sin i$, and $c = \cos i$. Here Ω is the node longitude of the orbit. Now, we write the potential given by equation (9) as a function of the orbital elements. Invoking equation (10) and the relation $\mu = n^2a^3$, we get

$$U_{20} = -\frac{1}{2}\frac{a^3}{r^3}\epsilon n^2(3s^2(\sin(f+g))^2-1) \tag{11}$$

where $\epsilon = J_2R_p^2$. Here n is the mean motion of the satellite. Analogously, for the sectorial perturbation (see reference [6]), we get

$$U_{22} = \frac{a^3}{r^3}\delta n^2(6\xi^2(\cos(f))^2 + 6\chi^2(\sin(f))^2 + 12\xi\chi\sin(2f) - 3 + 3s^2(\sin(f+g))^2) \tag{12}$$

where $\delta = C_{22}R_p^2$. To write the disturbing potential, we apply the single-averaged model. The development of the equations is carried out in closed form to avoid expansions in eccentricity and inclination. For this, it was necessary to perform algebraic manipulations where we used known equations of celestial mechanics, namely equations $a/r = (1 + e\cos(f))/(1-e^2)$ and $dl = \frac{1}{\sqrt{1-e^2}}\frac{r^2}{a^2}df$, where l is the mean anomaly of the spacecraft. After performing the single-averaged model over the true anomaly of the spacecraft using equation (11), and after some algebraic manipulations, we get

$$< R_{J2} >= -\frac{1}{4}\frac{\epsilon}{(1-e^2)^{3/2}}n^2(3s^2-2) \tag{13}$$

Now, using equation (12) and the ξ and χ variables to develop the potential due to the equatorial ellipticity of the planet, we get $< R_{C22} >= -\frac{3}{2}\frac{\delta}{(1-e^2)^{3/2}}n^2(c^2-1)\cos(2\Omega)$.

DOI: 10.5540/03.2018.006.02.0250

The approach to analyze the effect of the C_{22} term is based in references [2, 12]. In this equation, the node longitude of the orbit Ω is replaced by the expression $\Omega = h - \rho t$ given by reference [10], where ρ is the rotation rate of the planet and t is the time. Thus, by replacing this expression in the equation due to the C_{22} term, we get

$$< R_{C_{22}} >= -\frac{3}{2}\frac{\delta}{(1-e^2)^{3/2}} n^2(c^2-1)\cos(2\rho t - 2h) \tag{14}$$

4 Applications

The resonance of Kozai (see reference [8]) is a secular disturbing effect that causes variations in the orbital eccentricity and inclination and produces a libration (oscillation about a constant value) in the argument of the pericenter. The resonant potential considered in this application due to the Kozai resonance is given by equation (13) (oblateness-J_2) added to the secular terms and terms that are multiplied by $\cos(g)$ in equation (7). Using contour plots and a simplified disturbing potential a qualitative analysis for the motion is performed. All the periodic terms that are not factorized by $\cos(2g)$ are neglected. Following reference [7] a condition to investigate the existence of libration and circulation is given by: if $C > 0.6$ the motion is a circulation if $C < 0.6$ the motion can circulate and librate. Where the constant C is given by the formula $C = (1 - e^2)cos^2(i)$ (see reference [7]).

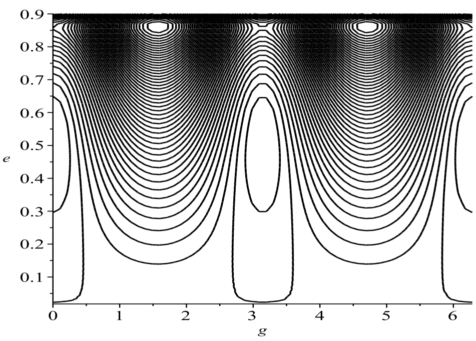

Figura 1: eccentricity (e) versus argument of the pericenter (g-rad). Disturbing potential: $R_{J_2} + R2_{SA_{Kozai}}$. Libration Center for C=0.05 and a=5000 km.

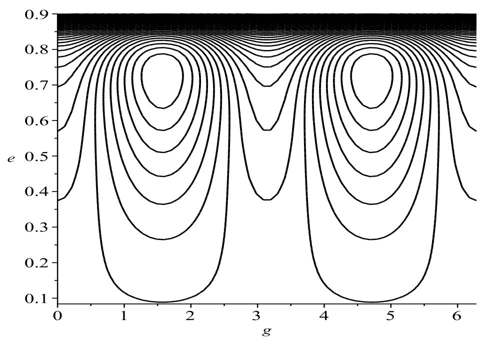

Figura 2: eccentricity (e) versus argument of the pericenter (g-rad). Disturbing potential: $J_2 + R2_{SA_{Kozai}}$. Libration Center for C=0.1 and a=5000 km.

The Figures 1 and 2 show the libration center of the orbits that librates around the equilibrium point, considering a constant C (see caption). In these two cases, the argument of the pericenter in the libration region is about $0°, 90°, 180°, 270°$, and these values are important in the case of frozen orbits (see reference [1]). For other values of the constant C we have circulation, where there is no libration center. The Figures 3 and 4 show that when the satellite is closest to the planet, some regions of libration have been destroyed and others amplified.

DOI: 10.5540/03.2018.006.02.0250

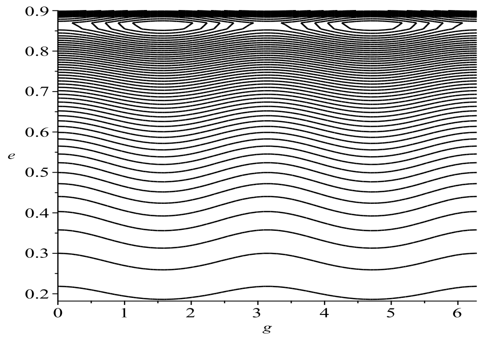

Figura 3: eccentricity (e) versus argument of the pericenter (g-rad). Disturbing potential: $R_{J_2} + R2_{SA_{Kozai}}$. Libration Center for C=0.05 and a=3000 km.

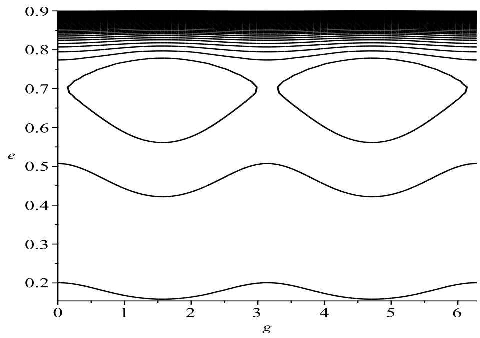

Figura 4: eccentricity (e) versus argument of the pericenter (g-rad). Disturbing potential: $J_2 + R2_{SA_{Kozai}}$. Libration Center for C=0.1 and a=3000 km.

5 Conclusions

The disturbing potential of single-averaged was developed considering the perturbations of the third body (in circular and planar orbit) and the non-sphericity of the central body. The two main terms due to flattening (J_2 and C_{22}) in the poles and in the equator, which are important in the case of the planet Mercury, were considered. With a simplified model for the disturbing potential, it is possible to make a qualitative analysis of the motion of artificial satellites. Considering our disturbing potential, we present an analysis with the resonant terms. Such terms are due to the resonance of Kozai. Regions are determined where the satellite cannot be found. These regions are defined by the Kozai resonance. The regions where the motion is of the libration type are important for an approach on studies of frozen orbits. This is of extreme importance for some types of space missions. In the continuity of this work, the disturbing potential due to the third body will be developed in elliptical orbit, since Mercury has a considerable eccentricity. Other terms of Mercury flattening will be taken into account in the disturbing potential (for example, the C_{22} and C_{31} terms). We will present a study considering the apsidal and evection resonances.

Acknowledgements

Sponsored by CNPq - Brazil. The authors are grateful to CNPq (National Council for Scientific and Technological Development) - Brazil for contracts 307724/2017-4, 420674/2016-0.

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