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Mathematical Modelling of Stress Effects to Predict High-Performance Production Spots in Oilfields

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In oil reservoirs, the overburden pressure (vertical stress) results from the cumulative weight of soil layers over other lower layers. Opposed to this force, the pressure caused by fluids that fill the pore space of the reservoir rocks acts to produce an effective stress. Understanding how this resulting net stress affects the compressibility curve of the rocks, change the porous medium's properties and cause volume variation is relevant to processes encompassing oil-in-place volume quantification, structural stability upkeep, production spot selection and well placement.

Several studies have been performed to detect sweet spots and define hydraulic flow units (HFU) in oilfields. Such regions generally are found from different techniques and quality indicators, such as the flow zone indicator (FZI) and the reservoir quality index (RQI). The HFU/FZI/RQI theoretical background is explained, for instance, in [2]. Recently, a few papers suggesting well placement strategies based on this theory in combination with graph centrality measures were published (see [1] and references). However, stress effects were disregarded by the authors. To close this gap, this work is intended to study porosity-permeability-stress models (from now on, $\phi - \kappa - \sigma$ models) that may lead to more appropriate indicators for prediction of high-performance production spots in oilfields undergoing depletion and, consequently, stress influences.

Moosavi et al. [3] have pointed out that estimating the pore volume compressibility (C_{pc}) is preferable to engineers instead resorting to rock sample assays due to pitfalls with their collection and in-lab manipulation. They have used the following relation between C_{pc} and σ:

$$C_{pc} = C_{pc}^{\infty} + \frac{\gamma}{\phi K} \exp\left(-\frac{\sigma}{K}\right), \tag{1}$$

where C_{pc}^{∞}, γ and K are parameters associated to the rock material.

The mathematical modelling assumes that the stress field will be computed over a cell-based discrete grid Ω containing clustered regions $C_{D,q}, D = 1, \dots, \delta, \quad q = 1, \dots, \varrho$ whose cells $\{w_q^i\}_{i=1}^{n_q}$ that form each cluster are associated to nodes $\{v_q^i\}_{i=1}^{n_q}$ of a graph $G_{D,q}$

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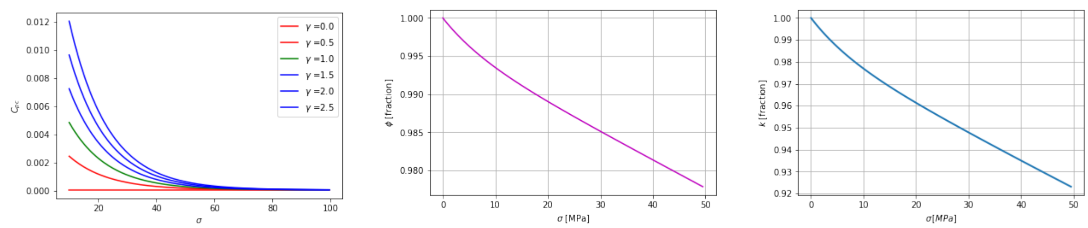
by a one-to-one mapping given as

$$\mathcal{F}\colon C_{D,q} \to G_{D,q}$$
$$w_q^i \mapsto v_q^i.$$

While previous works have considered a set of indicators derived only from ϕ (porosity) and k (permeability), among which the maximum closeness centrality (MCC) defined by

$$\max\{\gamma_{\phi,k}(v_q)\}|_D = \max_{1\leq i\leq n_q} \left\{ \frac{1}{\sum_i d(v_q, v_q^i)} \right\} \Bigg|_D \quad \forall D, q, \tag{2}$$

was a very significant one, we target to reach other indicators and a new MCC $\gamma_{\phi,k,\sigma}(v_q)$ which includes the stress field σ. In Figure 1, we plot several curves from Eq. 1 varying the parameter γ (a volume ratio) for a certain range of σ and taking values for a limestone sample in (a) and a $\phi-\kappa-\sigma$ model reproduced from [3] separated into two plots: in (b) and (c), we reproduce $\phi-\sigma$ and $k-\sigma$ models given, respectively, by $\phi = \frac{\exp(A)}{1+\exp(A)}$ and $k = \alpha\frac{\phi^3}{(1-\phi)^2}$ where ϕ_0 is an initial porosity, α a parameter depending on an initial permeability and the specific area of the porous cross section, and $A = -C_{pc}^{\infty}\sigma + \frac{\gamma}{\phi_0}(\exp(-\frac{\sigma}{K}) - 1) + \ln(\frac{\phi_0}{1-\phi_0})$.



(a) (b) (c)

Figure 1: Property changes in reservoir as a function of stress (a) $C_{pc} \times \sigma$ for several γ values; (b) $\phi-\sigma$ model; (c) $k-\sigma$ model.

We conclude that the stress effect over $\phi - k$ is highly dependent on C_{pc} and the rock material. Because of C_{pc} is usually constant in the modelling, a side effect on the search of the production spots is the shortened range of possible placement choices as outcomes.

References

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