Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 8, n. 1, 2021.

Trabalho apresentado no XL CNMAC, Evento Virtual - Co-organizado pela Universidade do Mato Grosso do Sul (UFMS).

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Graphs with few distinct eigenvalues and extremal energy

Nelcy E. Arévalo[1]
IME/UFRGS, Porto Alegre, RS
Rodrigo O. Braga[2]
IME/UFRGS, Porto Alegre, RS
Virgínia M. Rodrigues[3]
IME/UFRGS, Porto Alegre, RS

Among the various spectral parameters studied in the Spectral Graph Theory, one can highlight the *energy* of a graph, introduced by I. Gutman in 1978 [4]. The energy of a graph G with n vertices is defined as

$$\mathcal{E}(G) = \sum_{j=1}^{n} |\lambda_j|, \tag{1}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ are the eigenvalues of the adjacency matrix of G. Graph energy has been extensively studied (see for intance [7]).

Nikiforov noted in [9] and [10] that research on matrix norms is related to the study of graph energy. The *trace norm* $\|M\|_*$ of a matrix M is the sum of its singular values, which for a real symmetric matrix are exactly the modules of its eigenvalues. Hence the trace norm of the adjacency matrix $A = A(G)$ of a graph G is the energy of G, that is $\|A\|_* = \mathcal{E}(G)$.

Nikiforov in [10] gives bounds on the trace norm of matrices that extends previous results of Koolen and Moulton [6] for bounds on graph energy. In particular he gives a lower bound on the trace norm of matrices with rank at least 2. His bound gives a lower bound for the energy of a nonempty graph, i.e. a graph with at least one edge. In the same work he proposed the following problem, which is the aim of this work: give a constructive characterization of all graphs G such that the nonzero eigenvalues of G other than its largest eigenvalue have the same absolute value. Finding the solution of this problem would characterize the graphs that satisfy the equality in the lower bound for graph energy given by Nikiforov.

Furthermore, it is easy to see that if a graph satisfy the problem proposed by Nikiforov then it has at least 2 and at most 4 distinct eigenvalues. Graphs with few distinct eigenvalues form a largely studied class of graphs because they tend to have some interesting properties. A first nontrivial family of such graphs are the strongly regular graphs (the regular graphs with exactly three distinct eigenvalues), and we can consider other graphs with few distinct eigenvalues as a generalization of them. These graphs have been studied by several authors in many research papers (see for instance [2], [3], [5], [8], [11], [12], [13]).

The present work brings our results published in [1] in the study of the graphs that satisfy the problem proposed by Nikiforov.

[1]nearevalob@unal.edu.co
[2]rbraga@ufrgs.br
[3]vrodrig@mat.ufrgs.br

Acknowledgments

Nelcy Arévalo thanks CAPES for their support.

References

[1] Arévalo, N.E, Braga, R.O. and Rodrigues V.M. Graphs with few distinct eigenvalues and extremal energy, *Linear Algebra and its Applications*, 620:147-167, 2021. DOI: 10.1016/j.laa.2021.02.027.

[2] Bridges, W.G. and Mena, R.A. Multiplicative cones – a family of three eigenvalue graphs, *Aequationes Mathematicae*, 22:208–214, 1981. DOI: 10.1007/BF02190180.

[3] Chuang, H. and Omidi, G. Graphs with three distinct eigenvalues and largest eigenvalue less than 8, *Linear Algebra and its Applications*, 430:2053–2062, 2009. DOI: 10.1016/j.laa.2008.11.028.

[4] Gutman, I. The energy of a graph, *Ber. Math.– Statist. Sekt. Forschungsz. Graz*, 103:1–22, 1978.

[5] Huang, X. and Huang, Q. On regular graphs with four distinct eigenvalues, *Linear Algebra and its Applications*, 512:219–233, 2017. DOI: 10.1016/j.laa.2016.09.043.

[6] Koolen, J. and Moulton, V. Maximal energy graphs, *Advances in Applied Mathematics*, 26:47–52, 2001. DOI: 10.1006/aama.2000.0705.

[7] Li, X., Shi, Y. and Gutman, I. *Graph Energy.* Springer, New York, 2012.

[8] Muzychuk, M. and Klin, M. On graphs with three eigenvalues, *Discrete Mathematics*, 189:191–207, 1998. DOI: 10.1016/S0012-365X(98)00084-3.

[9] Nikiforov, V. The energy of graphs and matrices, *Journal of Mathematical Analysis and Applications*, 326:1472–1475, 2007. DOI: 10.1016/j.jmaa.2006.03.072.

[10] Nikiforov, V. Beyond graph energy: Norms of graphs and matrices, *Linear Algebra and its Applications*, 506:82–138, 2015. DOI: 10.1016/j.laa.2016.05.011.

[11] Van Dam, E.R. Regular graphs with four eigenvalues, *Linear Algebra and its Applications*, 226-228:139–162, 1995. DOI: 10.1016/0024-3795(94)00346-F.

[12] Van Dam, E.R. Graphs with few eigenvalues. An interplay between combinatorics and algebra, Doctoral Thesis, Tilburg University, 1996.

[13] Van Dam, E.R. Nonregular graphs with three eigenvalues, *Journal of Combinatorial Theory, Series B,*, 73:101–118, 1998. DOI: 10.1006/jctb.1998.1815.